Modèles mathématiques
Catia V5

http://www.diderot.org/

61, rue David d'Angers
75019 PARIS

SOMMAIRE

Contexte du travail présenté

Etude des modèles sur Catia V5

- **Courbes**
- **Surfaces**
- **Analyses**

Prochains Objectifs

Conclusions

Remarques préliminaires :

- Travail non terminé !!
- Document Franco-Anglais
- Valider sur certains points par D. S.

Formations, diplômes préparés
Lycée Diderot

Les diplômes et métiers préparés

- > CPGE-ATS (Vers le métier d'ingénieur)
- > Les licences Pro (Bachelors)
- > Les B.T.S industriels
- > Les bacs S.T.I (Sciences et Tech Indust)
- > Le bac S (option Sciences de l'ingénieur)
- > Les bacs Professionnels
- > Le B.E.P MPMI
- > Le C.A.P Horlogerie

Les spécialités enseignées

- > Electronique
- > CPI - Conception de Produits Industriels
- > Electrotechnique
- > IRIS (Info industrielle)
- > CIM - Microtechniques
- > MAI - Maintenance et Autom. Industriels
- > Traitement des Matériaux

- **Microtechniques**
- **E.D.P.I : Etude et définition de produits industriels**
- **Artisanat et métiers d'art option : Horlogerie**

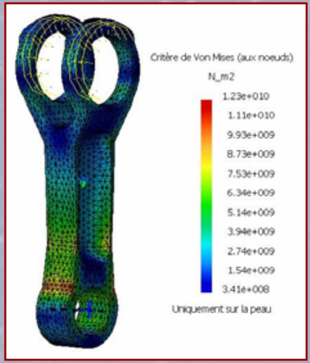
Les voies d'accès

- > Formation initiale (Bac et BTS)
- > Apprentissage (BTS)
- > Formation continue (BTS)

http://www.gita.greta.fr/

Licence Professionnelle « Métiers de la production »
option CONCEPTION COLLABORATIVE – MAQUETTE VIRTUELLE

Partenariat avec l'IUT de Saint Denis
En contrat d'apprentissage



Gestion de la connaissance

Conception de Produit

Conception et Calculs

SMARTEAM

Travail Collaboratif

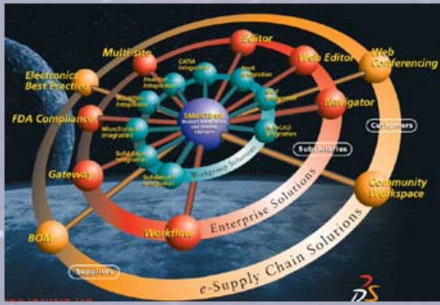
Conception Forme de style

Maquette Numérique

CATIA V5

Gestion de données techniques

Conception et Fabrication



Utiliser le potentiel de Catia V5 peu utilisé en BTS:
- Generative Shape Design et Freestyle
- Knowledge Advisor, Expert ….
- Generative Structural Analysis

Licence Professionnelle

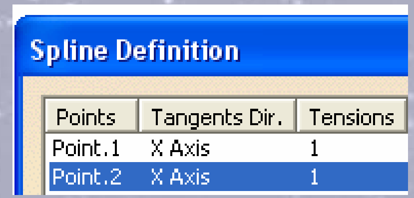
CONCEPTEUR NUMERIQUE EN DESIGN ET TECHNIQUE AUTOMOBILE

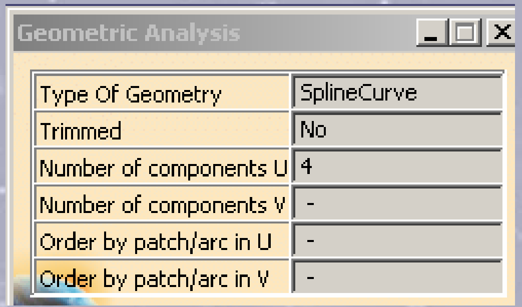
Demandeurs PSA et Renault

Début septembre 2006

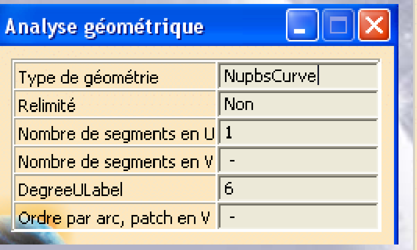
Définition de surfaces Class A à partir de courbes de Bézier

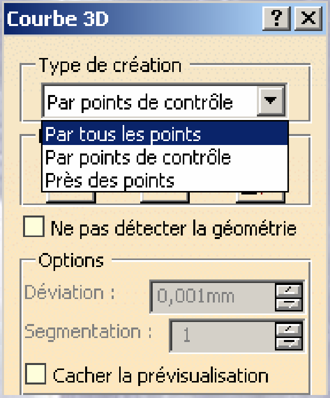
OBJECTIFS :





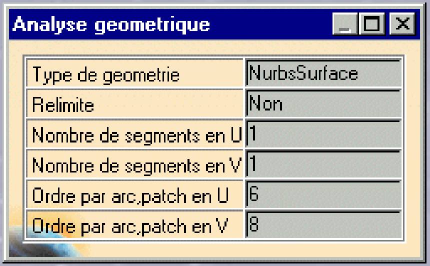
- Formation de niveau BAC + 3
- Comprendre les informations données par le logiciel
- Appliquer les notions mathématiques en manipulant sur Catia V5





Type of geometry posted	Definition
NupbsCurve	Curve NUPBS no rational
NupbsSurface	Surface NUPBS no rational
NurbsCurve	Curve NURBS rational
NurbsSurface	Surface NURBS rational
PNupbs	Parametric Curve rational
PNurbs	Parametric Curve on surface
PSpline	Parametric Curve

Evolution jusqu'aux NURBS


Analyse geometrique

Type de geometrie	NurbsSurface
Relimite	Non
Nombre de segments en U	1
Nombre de segments en V	1
Ordre par arc,patch en U	6
Ordre par arc,patch en V	8

Modeleur paramétrique variationnel

PARAMETRAGE

NURBS

POLYNOMIALS CURVES

CUBIC - QUINTIC SPLINE

NO UNIFORM B-SPLINE

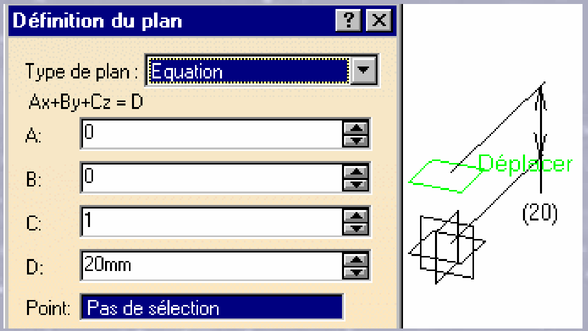
BEZIER CURVE

B-SPLINE

RATIONAL BEZIER CURVE

Representations

Il n'y a pas que la représentation paramétrique



Explicit representation

Explicit form of a curve 2D : y = f(x)

Explicit form of a surface : z = f(x, y)

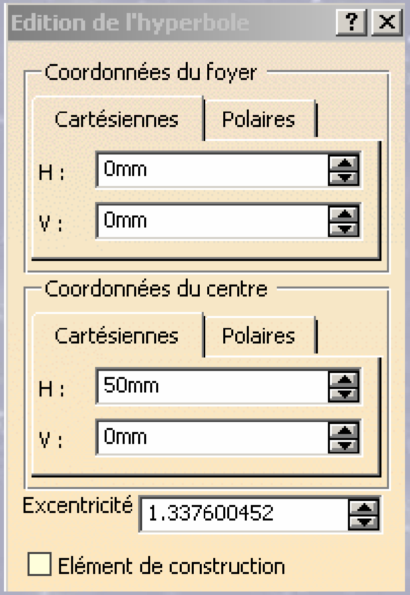
Implicit representation

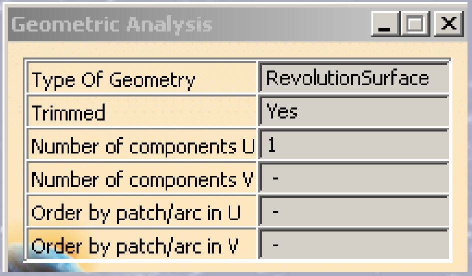
Implicit form of a curve 2D: f(x, y) = 0

Implicit form of a surface 3D: f(x, y, z) = 0

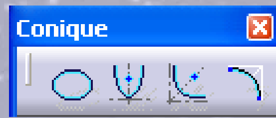
Parametric représentation

- Curve x(u), y(u), z(u)
- Surface x(u, v), y(u, v), z(u, v)



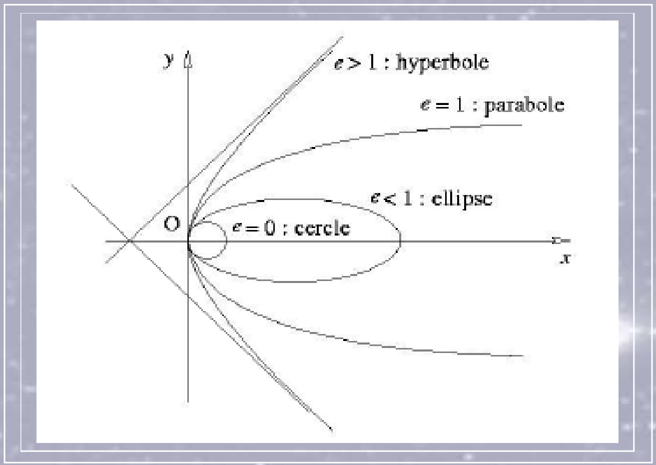


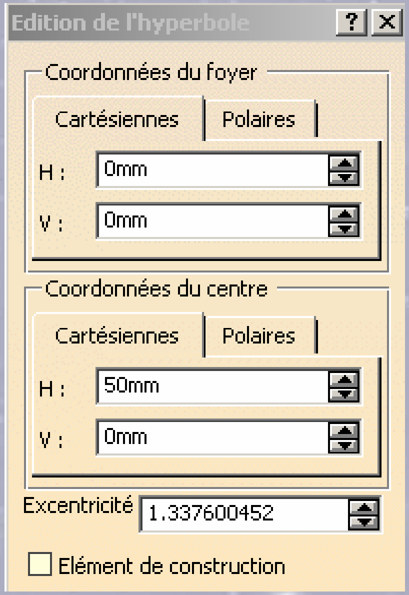
CONIQUES



Implicite equation

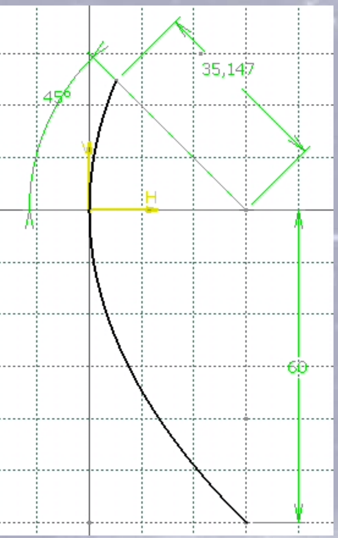
$$y^2 - 2.p\,x - (1 - e^2)\,x^2 = 0$$

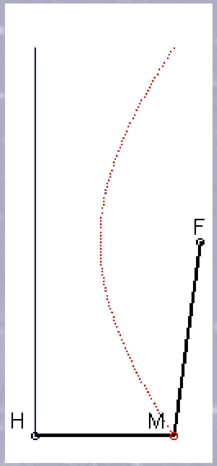




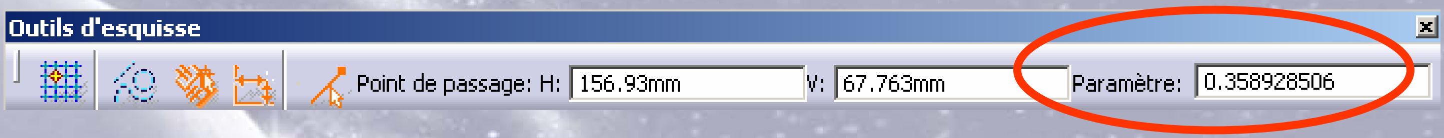
Explicit equation

$$\rho = \frac{p}{1 + e\cos\theta}$$





Parametric equation ???



0 < Parameter < 0,5 → Ellipse

Parameter = 0,5 → Parabola

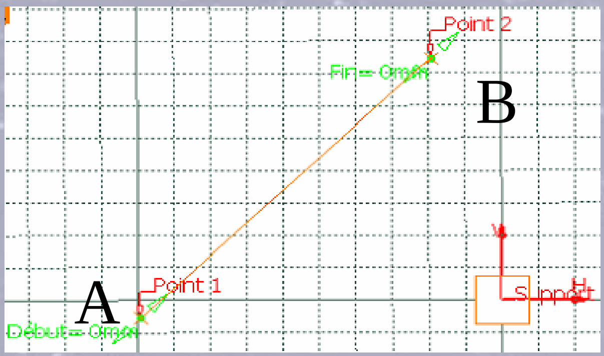
0,5 < Parameter < 1 → Hyperbola

PARAMETRE ≠ EXCENTRICITE

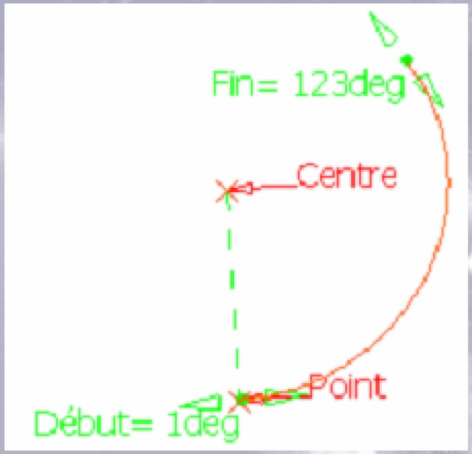
Paramétrage : Droite - Cercle

$$\overrightarrow{D}(u) = (1 - u)\ \overrightarrow{OA} + u\ \overrightarrow{OB}$$

u parameter varying from 0 to 1



$$\overrightarrow{A}(u) = R\,(\cos U\ \overrightarrow{e_1} + \sin U\ \overrightarrow{e_2}) + \overrightarrow{OC}$$



Paramétrage en abscisse curviligne

$$\overrightarrow{OM} = \overrightarrow{OA} + s\ \frac{\overrightarrow{AB}}{\left\|\overrightarrow{AB}\right\|}$$

With $0 < s < \left\|\overrightarrow{AB}\right\|$

$$\overrightarrow{A}(s) = R\,(\cos(s/R)\ \overrightarrow{e_1} + \sin(s/R)\ \overrightarrow{e_2}) + \overrightarrow{OC}$$

AVANTAGES

Trièdre de Frenet

$$\frac{d}{ds}\begin{bmatrix} T \\ N \\ B \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix}$$

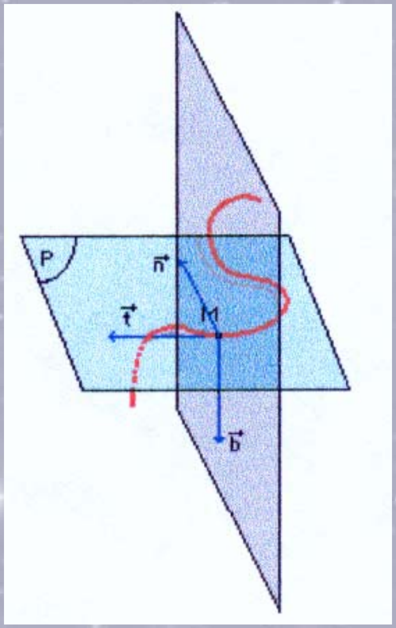
R_c : curvature radius

R_t : torsion radius

κ : curvature

τ : torsion

$$\vec{B} = \vec{T} \wedge \vec{N}$$

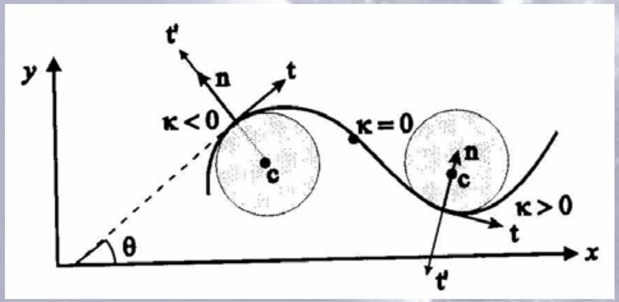


$$\frac{d\vec{T}}{ds} = \frac{\vec{N}}{R_c}$$

$$\frac{d\vec{B}}{ds} = \frac{\vec{N}}{R_t}$$

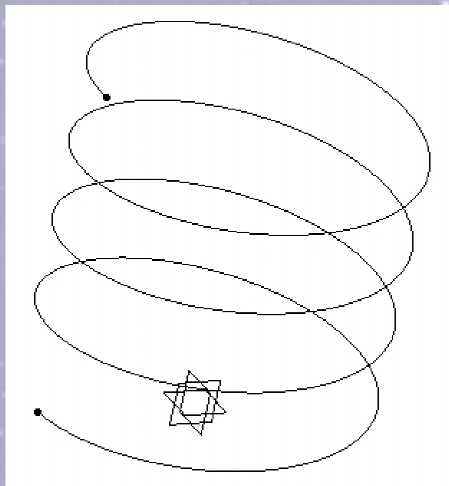
$$\left\|\vec{N}\right\| = 1$$

$$\left\|\frac{d\overrightarrow{OM}^2}{ds^2}\right\| = 1/R_c$$



Avec un paramétrage en abscisse curviligne, il est plus facile de déterminer les rayons de courbure d'une courbe 3D

Curvature – torsion helix



$$\vec{T} \left| \begin{array}{l} x = -R/c \sin(s/c) \\ y = R/c \cos(s/c) \\ z = p/c \end{array} \right. \qquad \vec{N} \left| \begin{array}{c} -\cos(s/c) \\ -\sin(s/c) \\ 0 \end{array} \right.$$

$$\vec{T} \cdot \vec{N} = 0$$

$$\vec{B} = \vec{T} \wedge \vec{N}$$

R = 20 mm

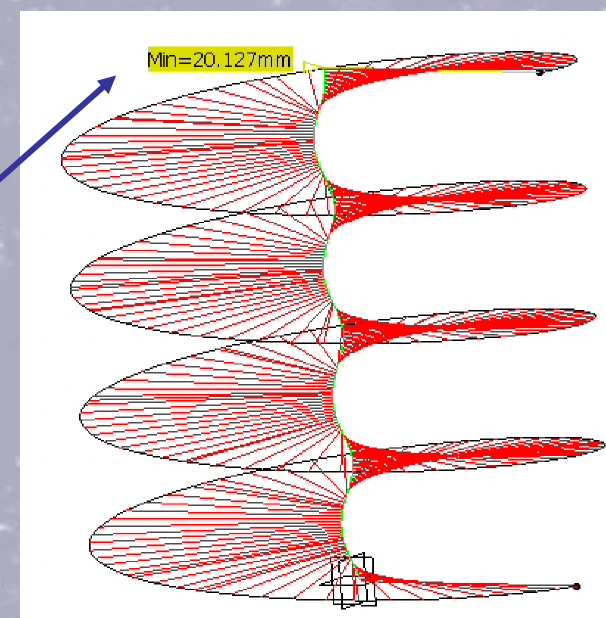
$p = 10 / 2\pi$

$$\vec{dT/ds} \left| \begin{array}{c} -R/c^2 \cos(s/c) \\ -R/c^2 \sin(s/c) \\ 0 \end{array} \right.$$

$$c = \sqrt{R^2 + p^2}$$

Curvature radius

$Rc = (R^2 + p^2) / R$



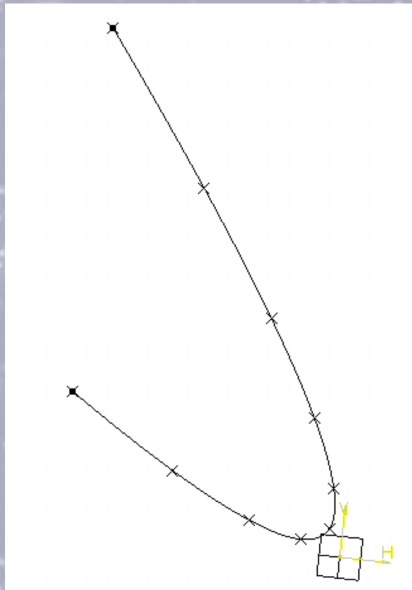
$$\vec{B} \left| \begin{array}{c} p/c \sin(s/c) \\ -p/c \cos(s/c) \\ R/c \end{array} \right.$$

Torsion radius

$Rt = -(R^2 + p^2) / p$

$$\vec{dB/ds} \left| \begin{array}{c} p/c^2 \cos(s/c) \\ p/c^2 \sin(s/c) \\ 0 \end{array} \right.$$

Rt = 252,9 mm



Example 2 :

$-4 < u < 5$

$$\mathcal{C}(u) \left| \begin{array}{l} x = u^2 + u + 1 \\ y = u^2 - 2u + 2 \end{array} \right.$$

$$ds/du = \sqrt{8u^2 - 4u + 5}$$

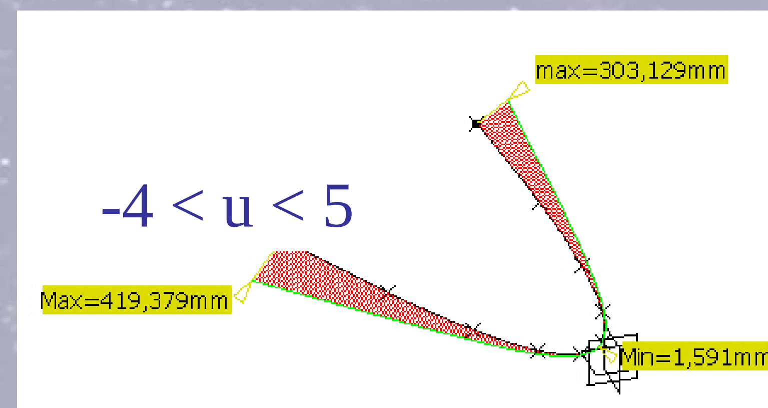
$$\vec{T} = d\,\vec{OM}/du \times du/ds = \left| \begin{array}{l} (2u+1)\,du/ds \\ 2(u-1)\,du/ds \end{array} \right.$$

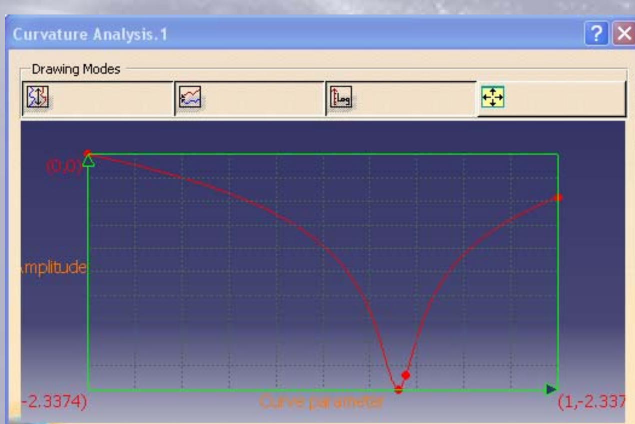
$$\vec{N} = \frac{d\vec{T}/ds}{\left\| d\vec{T}/ds \right\|}$$

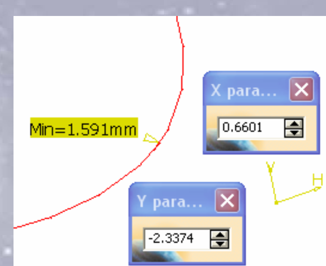
$$\vec{N} = \left| \begin{array}{l} -2(u-1)\,du/ds \\ (2u+1)\,du/ds \end{array} \right.$$

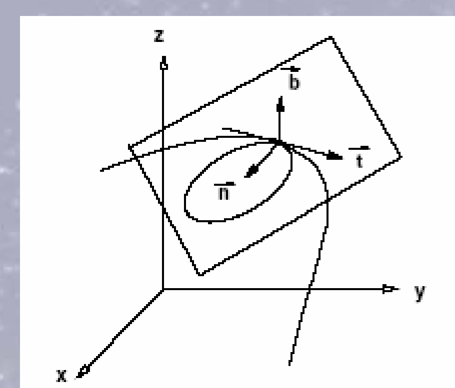
$$\frac{d\vec{T}}{ds} = \frac{\vec{N}}{R_c} \longrightarrow$$

$$1/R_c = \frac{6}{(8u^2 - 4u + 5)^{3/2}}$$



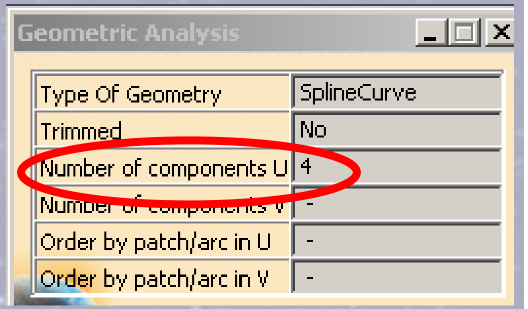




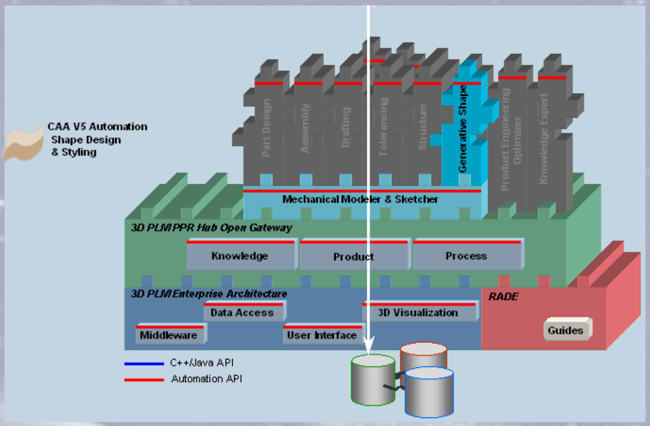


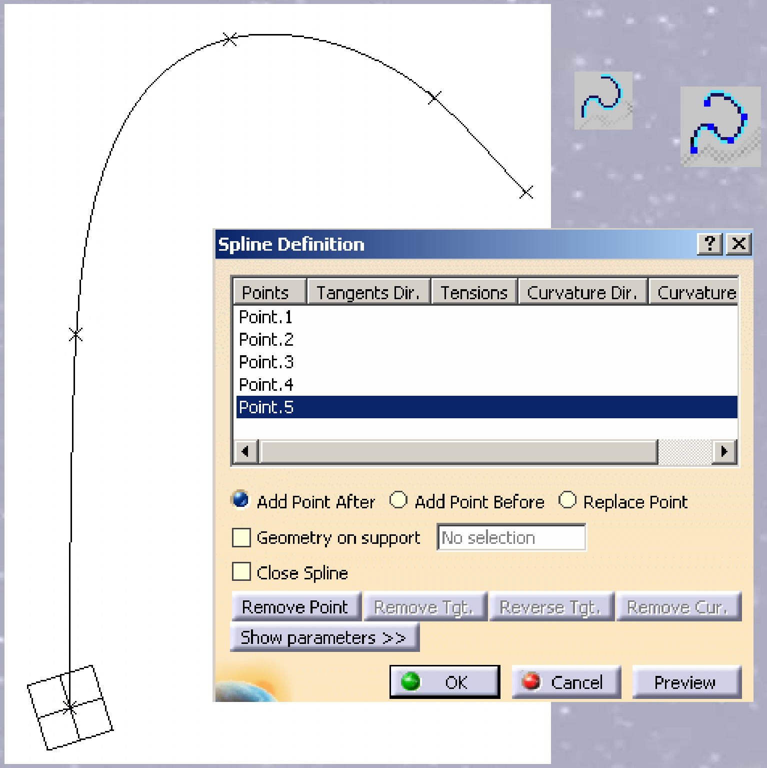
SPLINE

SPLINE CUBIQUE OU QUINTIQUE ??



Courbe polynomiale
par morceaux





Sub **SetSplineType**(long iSplineType)

Sets the spline type.

Parameters: iSplineType

The spline type

Legal values: Cubic spline or WilsonFowler ????

SPLINE CUBIQUE

$$\overrightarrow{OM} = (2u^3 - 3u^2+1)\,\overrightarrow{OP_0} + (-2u^3 + 3u^2)\,\overrightarrow{OP_1} + (u^3 - 2u^2 + u)\,\overrightarrow{dOP_0/du} + (u^3 - u^2)\,\overrightarrow{dOP_1/du}$$

$$\vec{P}(u) = (x, y, z) = [u^3, u^2, u^1, 1] \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x(0) & y(0) & z(0) \\ x(1) & y(1) & z(1) \\ x'(0) & y'(0) & z'(0) \\ x'(1) & y'(1) & z'(1) \end{bmatrix}$$

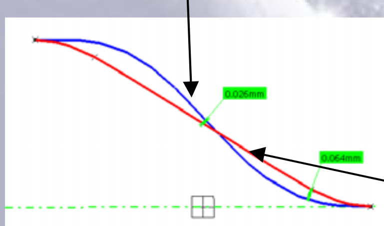
SPLINE QUINTIQUE

$$\overrightarrow{OM} = H_1(u)\,\overrightarrow{OP_0} + H_2(u)\,\overrightarrow{OP_1} + H_3(u)\,\overrightarrow{dOP_0/du} + H_4(u)\,\overrightarrow{dOP_1/du} + H_5(u)\,\overrightarrow{d^2OP_0/du^2} + H_6(u)\,\overrightarrow{d^2OP_0/du^2}$$

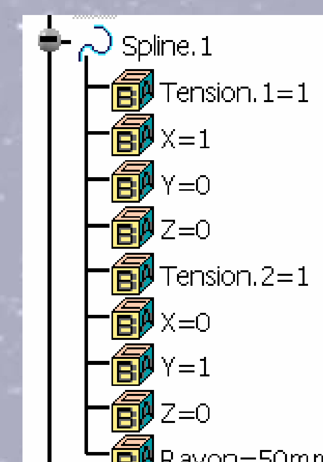
$$M(u) = [u][M][P]$$

$$[M] = \begin{pmatrix} -6 & +6 & -3 & -3 & -1/2 & +1/2 \\ -15 & -15 & +8 & +7 & +1,5 & -1 \\ -10 & +10 & -6 & -4 & -1,5 & +1/2 \\ 0 & 0 & 0 & 0 & 0,5 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

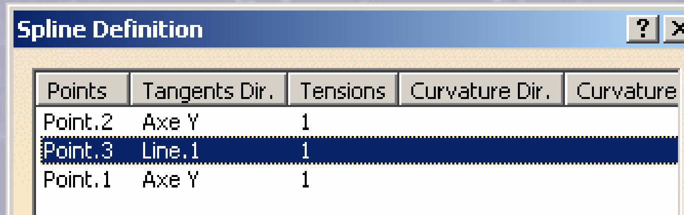
Spline with Catia



Spline quintic calculed

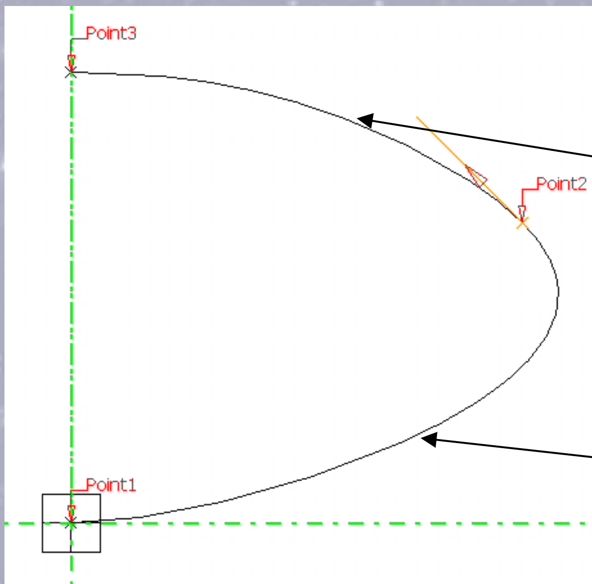


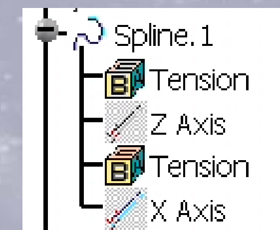
TENSION



Point 1 (0,0,0); Point (3,2,0); Point 3 (0,3,0)

Line at the point 2 : angle 45°





$\mathcal{C}_2(u)$

$$x = 3 - u - 6u^2 + 4u^3$$
$$y = 2 + u + u^2 - u^3$$

$\mathcal{C}_1(u)$

$$x = u + 8u^2 - 6u^3$$
$$y = 5u^2 - 3u^3$$

$$M(u) = [u][M][P]$$

$$\mathcal{C}_1(u) \Longrightarrow [M][P] = \begin{pmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 3 & 2 & 0 \\ 1 & 0 & 0 \\ -1 & 1 & 0 \end{pmatrix} = \begin{bmatrix} -6 & -3 & 0 \\ 8 & 5 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

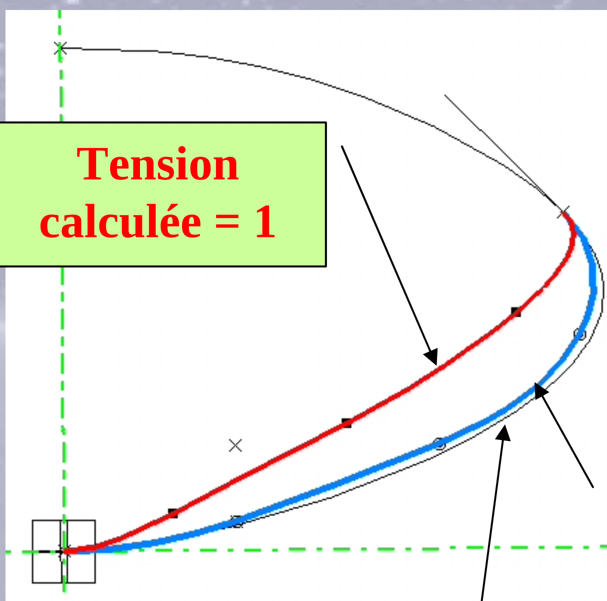
$$\begin{pmatrix}2&-2&1&1\\-3&3&-2&-1\\0&0&1&0\\1&0&0&0\end{pmatrix}\begin{pmatrix}0&0&0\\3&2&0\\3&0&0\\-3&3&0\end{pmatrix} = \begin{bmatrix}-6&-1&0\\6&3&0\\3&0&0\\0&0&0\end{bmatrix}$$

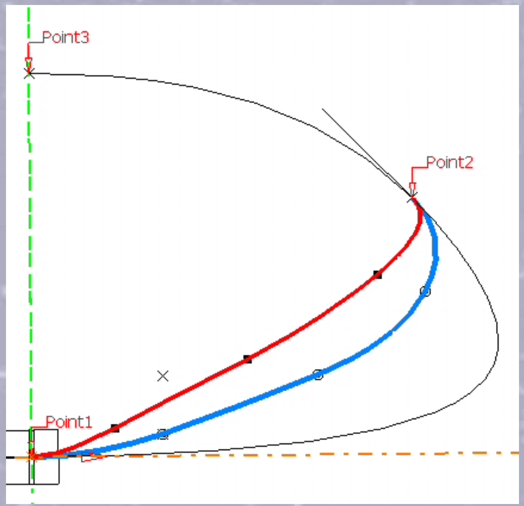
$\mathcal{C}_1(u)$ become

$$X = 3u + 6u^2 - 6u^3$$
$$y = 3u^2 - u^3$$

Tension s

$$\begin{pmatrix}0&0&0\\3&2&0\\s&0&0\\-s&s&0\end{pmatrix}$$

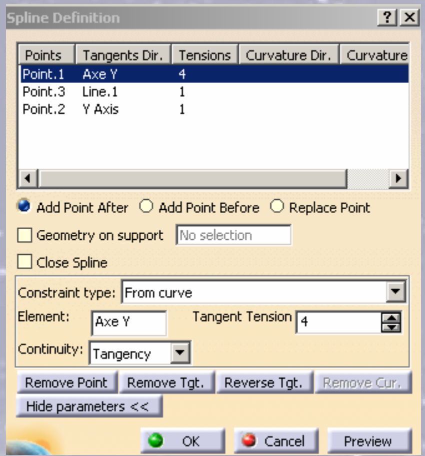




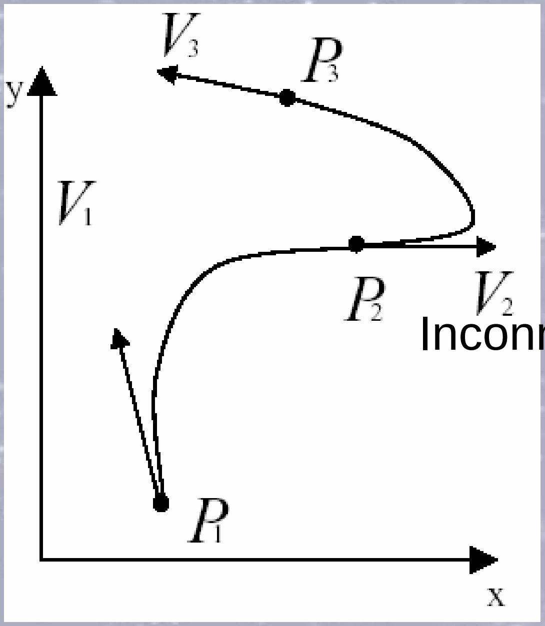
Tension calculée = 1

Tension calculée = 3

Pb : Tension Catia = 1



Boundary Conditions



Inconnue

Une spline passant par trois points et dont les tangentes V1 et V3 sont connues

Deux courbes de degré 3 dans le **plan XY**

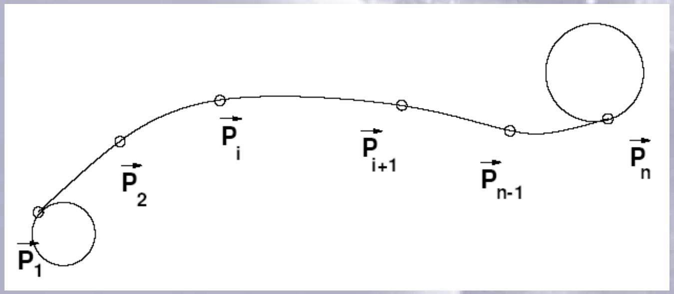
$\rightarrow$ 2 × 8 =16 unknown factors

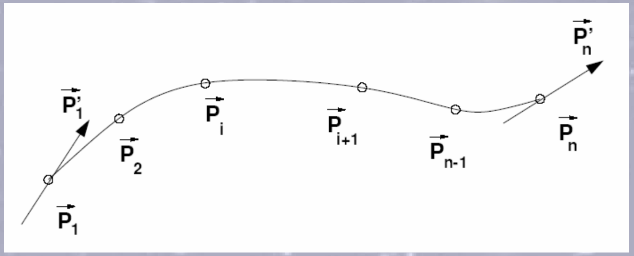
Three points (2×4) + Condition of tangency and curvature between the 2 curves $\rightarrow$ 12 Equations

It is necessary to add 4 conditions

- To impose the tangency at the ends of the curve
- To impose a curvature given at the two ends

(Null curvature $\rightarrow$ natural Cubic Spline)





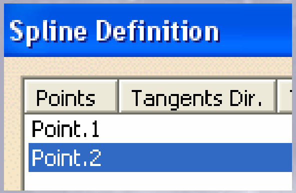
Least Energy

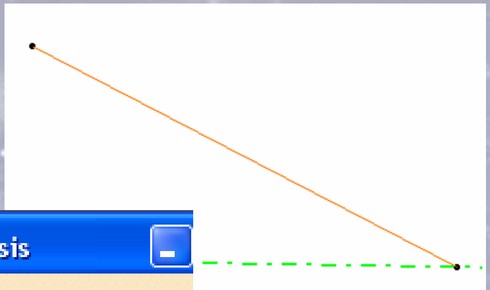
Minimizes the internal strain energy
so the minimum curvature

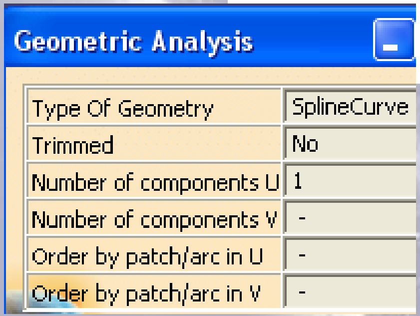
$$\delta = \int \kappa^2 ds$$ minimized

κ = curvature

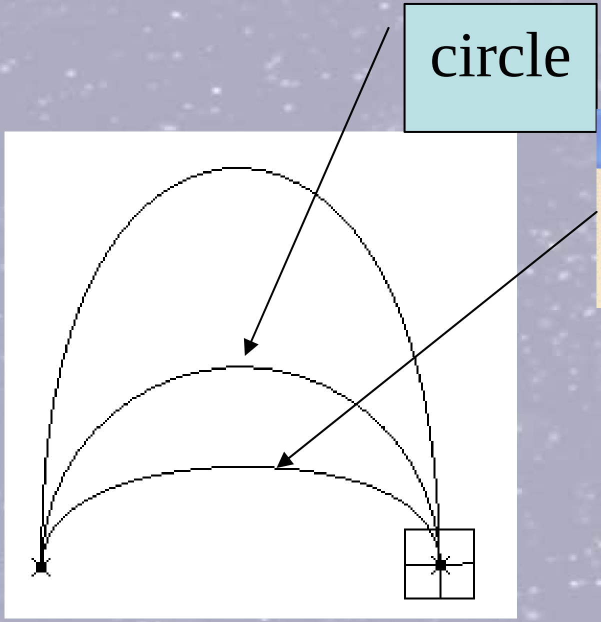
s = curvilinear abscissa

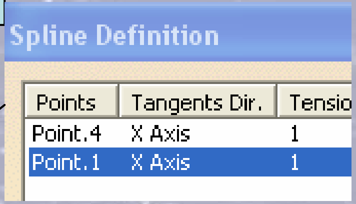






Type Of Geometry	SplineCurve
Trimmed	No
Number of components U	1
Number of components V	-
Order by patch/arc in U	-
Order by patch/arc in V	-

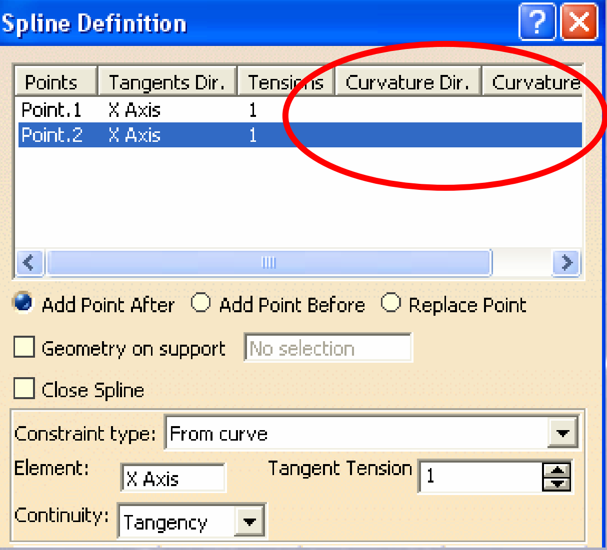




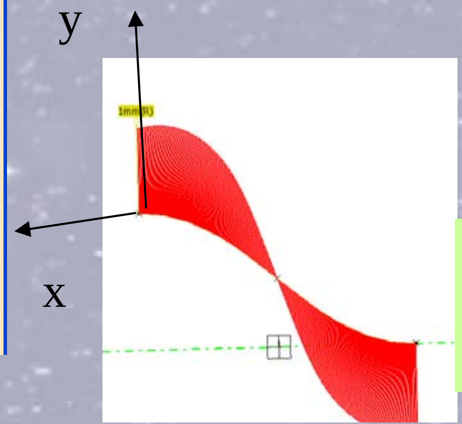
4 constraints

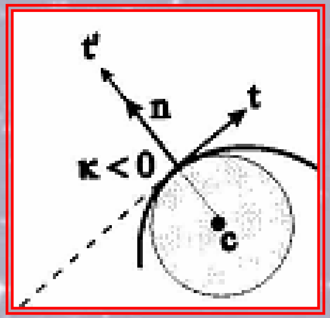
Spline degree 5

⇒ **Least energy**



Curve with $\frac{\vec{N}}{Rc} = \begin{vmatrix} 0 \\ +1 \\ 0 \end{vmatrix}$

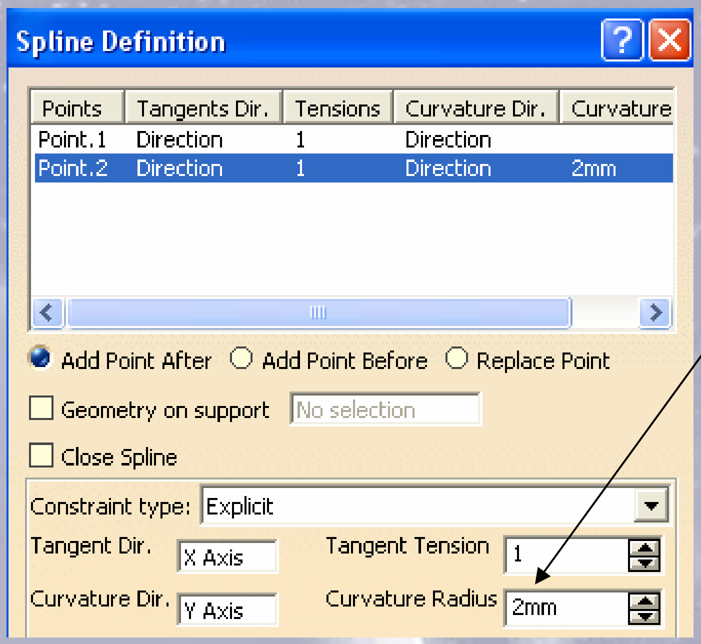


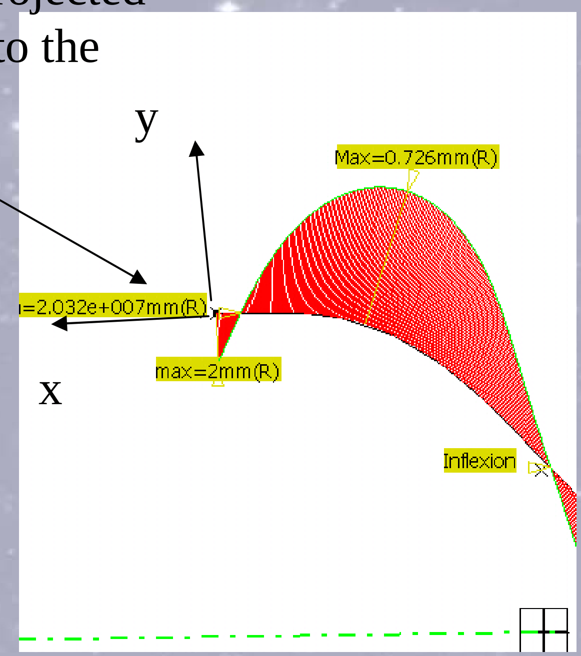


What solution to draw the curve without all the conditions ?

The curvature direction is projected on a plane perpendicular to the tangency direction

$$\frac{\vec{N}}{Rc} = \begin{vmatrix} 0 \\ -1/2 \\ 0 \end{vmatrix}$$





BEZIER CURVE

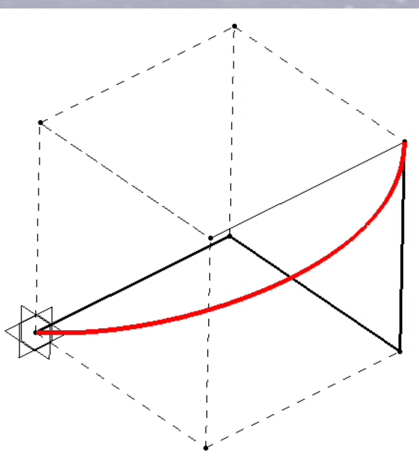
Limits of splines

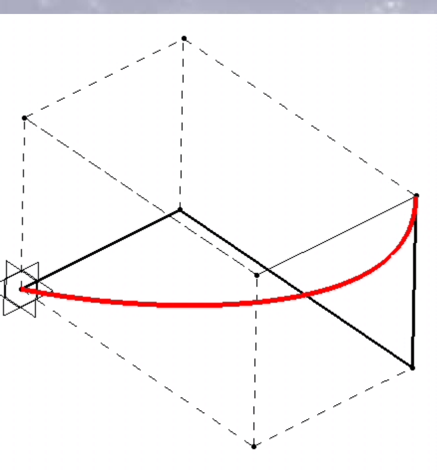
The numerical definition of a Bézier curve

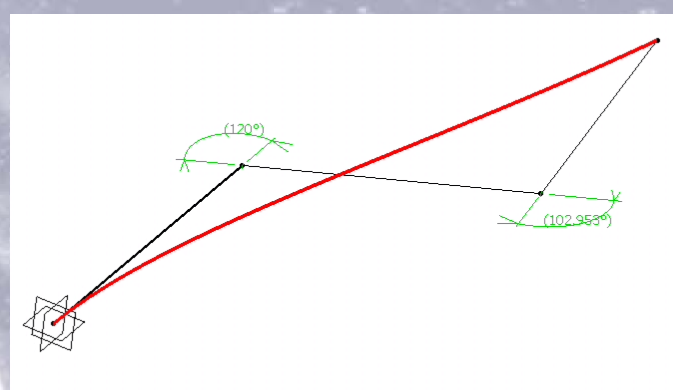
Take a cube and draw a curve, intersection of two cylinders

BÉZIER Pierre 1910-1999

$$\overrightarrow{OM} = \sum_{k=0}^{n} B_{k,n}(u)\overrightarrow{OA_k}$$







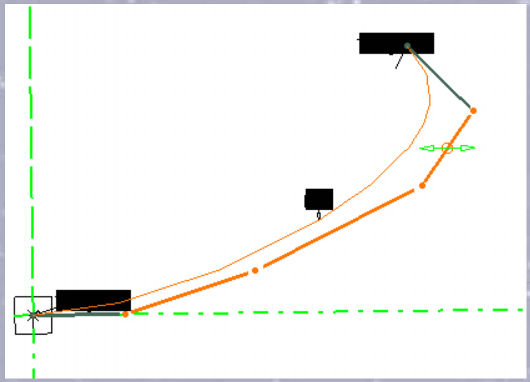
$$P(u) = UMB^T = [u^3 \; u^2 \; u \; 1] \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P1 \\ P2 \\ P3 \\ P4 \end{bmatrix}$$

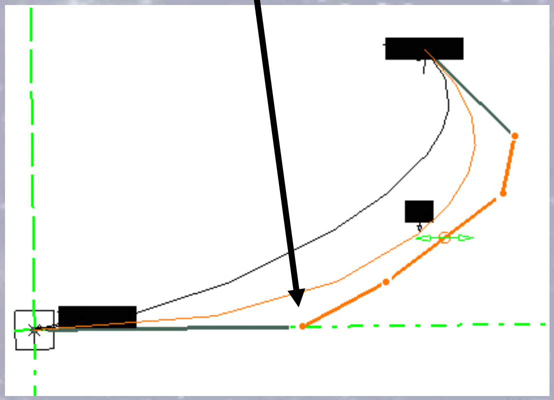
Tangency and Curvature

$$\overrightarrow{\mathbf{T}} = \frac{\frac{d}{du}\overrightarrow{OM(u)}}{\left\| \frac{d}{du}\overrightarrow{OM(u)} \right\|}$$

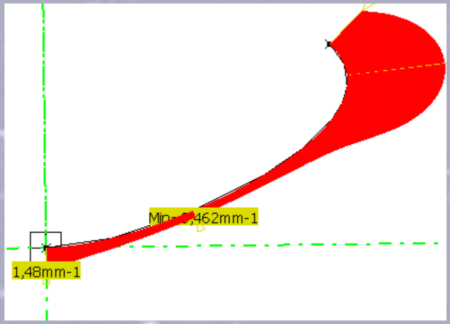
Displacement of the second pole horizontally (according to the direction of tangency at the point)

$$\overrightarrow{T(u=0)} = \frac{\overrightarrow{OP_1} - \overrightarrow{OP_0}}{\left\| \overrightarrow{OP_1} - \overrightarrow{OP_0} \right\|}$$





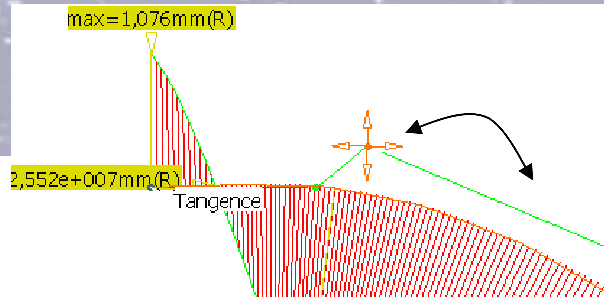
$$\kappa = \frac{\left\| \left(\frac{d}{dt}\overrightarrow{OM}\right) \wedge \left(\frac{d^2}{dt^2}\overrightarrow{OM}\right) \right\|}{\left\| \frac{d}{dt}\overrightarrow{OM} \right\|^3}$$

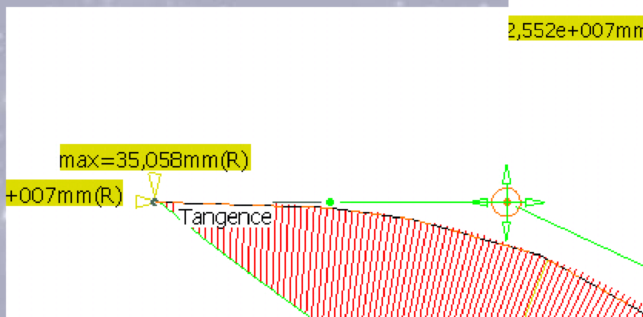


We can write

$$\frac{d^2}{du^2}\left[\overrightarrow{OM(0)}\right] = n \cdot n-1 \cdot \left[\left(\overrightarrow{OP_0} - \overrightarrow{OP_1}\right) - \left(\overrightarrow{OP_1} + \overrightarrow{OP_2}\right)\right]$$

Tangency



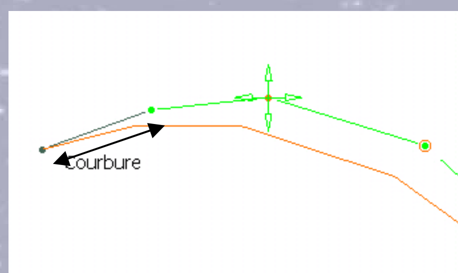


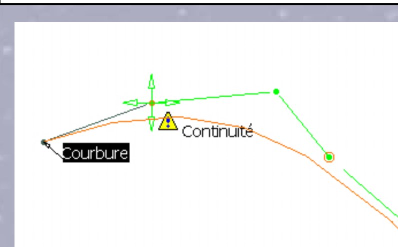
$$\vec{T}(u=0) = \frac{\overrightarrow{OP_1} - \overrightarrow{OP_0}}{\left\| \overrightarrow{OP_1} - \overrightarrow{OP_0} \right\|}$$

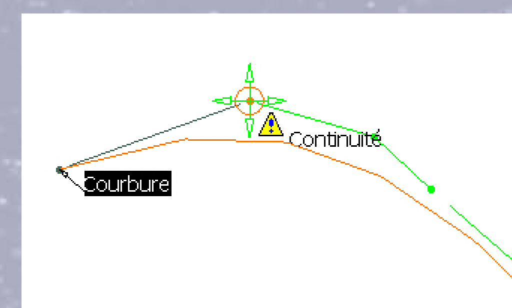
Displacement of the second pole horizontally (according to the direction of tangency at the point).
You can change the position of the third point, as you want

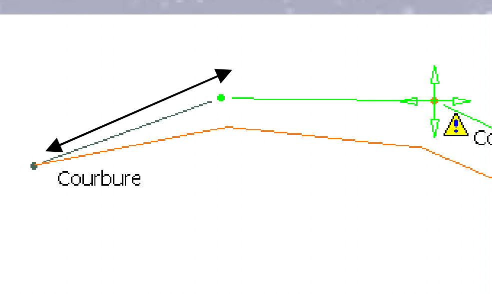
Curvature

Curvature(u=0)= $$\frac{n-1 \left\| (\overrightarrow{OP}_0 - \overrightarrow{OP}_1) \wedge (\overrightarrow{OP}_2 - \overrightarrow{OP}_1) \right\|}{n \left\| \overrightarrow{OP}_1 - \overrightarrow{OP}_0 \right\|^3}$$









You can change the position of the third point but :

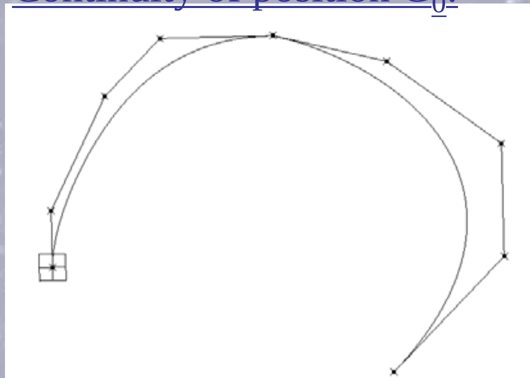
not in all direction

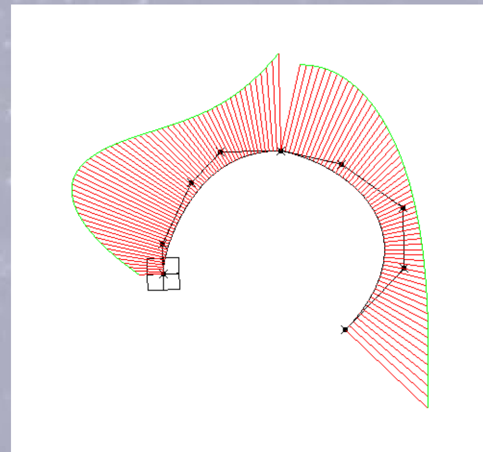
with a ratio between the 3 first points

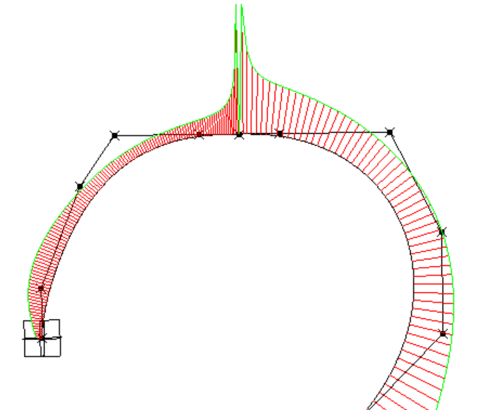
The n-th derivative of a bezier curve for one as of the his ends depends only on the n+1 points of control nearest to this end

With Catia

Continuity of position G_0:

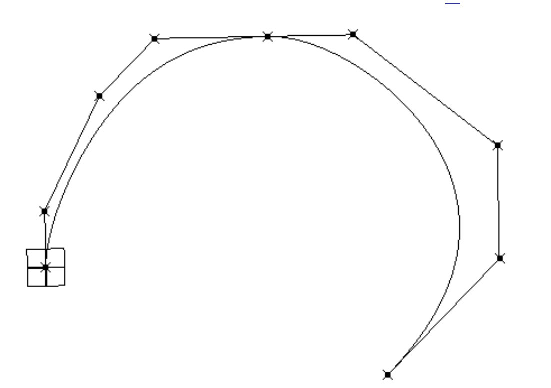


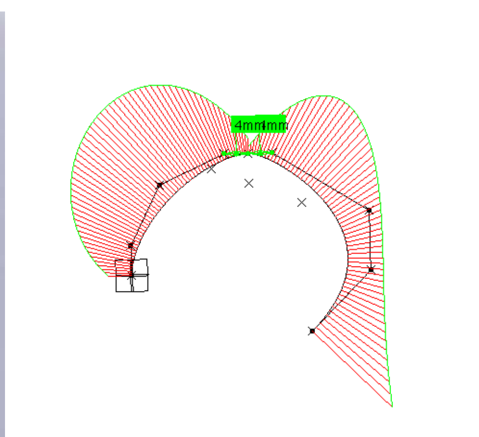


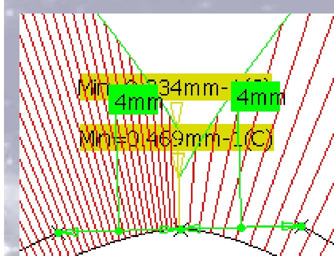


3 points aligned ⇒ curvature = 0

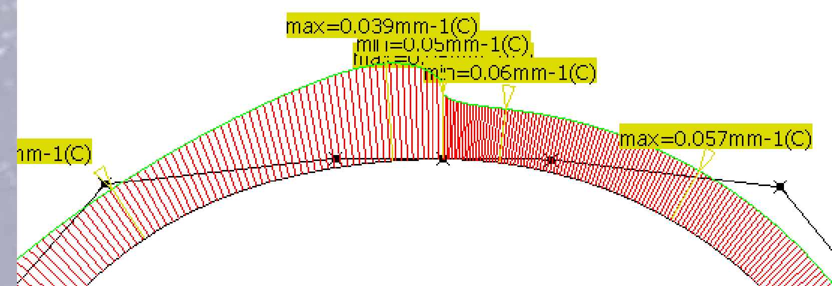
Continuity of class G_1 :







3 points no aligned ⇒ curvature ≠ 0



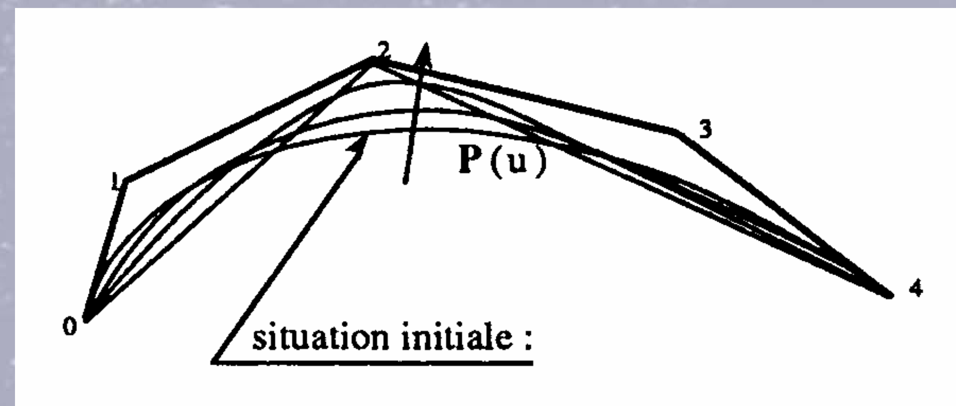
$$\textbf{Curvature(u=0)} = \frac{n-1\left\| (\overrightarrow{OP_0}-\overrightarrow{OP_1}) \wedge (\overrightarrow{OP_2}-\overrightarrow{OP_1}) \right\|}{n\left\| \overrightarrow{OP_1}-\overrightarrow{OP_0} \right\|^3}$$

Rational form of the Bezier 's arc

$$\overrightarrow{OM} = \frac{\sum_{i=0}^{m} B_m^i(u)\, \lambda_i\, \overrightarrow{OP_i}}{\sum_{j=0}^{m} \lambda_i\, B_m^i(u)}$$

The parameter λ_i acts like a weight on the associated top by attracting the curve towards this top.

Influence of λ_i :

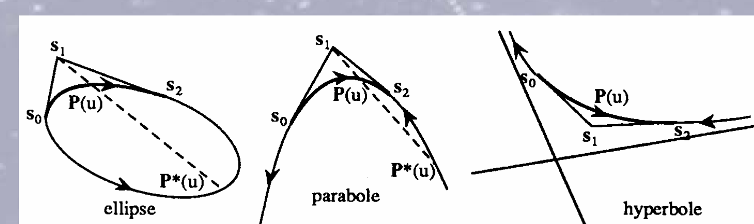


$\lambda_i \rightarrow \infty$

Three tops P_0, P_1, P_2 define the characteristic polygon of the curve of degree 2.

$$\overrightarrow{OM} = \frac{\lambda_0 B_{0,2}\overrightarrow{OP_0} + \lambda_1 B_{1,2}\overrightarrow{OP_2} + \lambda_2 B_{2,2}\overrightarrow{OP_2}}{\lambda_0 B_{0,2} + \lambda_1 B_{1,2} + \lambda_2 B_{2,2}}$$

$$= \frac{\lambda_0 (1-u)^2\overrightarrow{OP_0} + 2\lambda_1 u(1-u)\overrightarrow{OP_1} + \lambda_2 u^2\overrightarrow{OP_2}}{\lambda_0 (1-u)^2 + 2\lambda_1 u(1-u) + \lambda_2 u^2}$$



$$\overrightarrow{OM}=\frac{\lambda_0(1-u)^2\overrightarrow{OP_0}+2\lambda_1 u(1-u)\overrightarrow{OP_1}+\lambda_2 u^2\overrightarrow{OP_2}}{\lambda_0(1-u)^2+2\lambda_1 u(1-u)+\lambda_2 u^2}$$

If $\lambda_0 = 1-\lambda$; $\lambda_1=\lambda$; $\lambda_2=1-\lambda$,

the denominateur become $= 2(1-2\lambda)u^2-2(1-2\lambda)u+1-\lambda$

the discriminant Δ for finding the roots of the denominateur is $= 2\lambda - 1$

When $\lambda > ½$, the denominator has different real roots, the curve have asymptotes

When $\lambda < ½$, the denominator does not have any real roots, the curve does not have an asymptote

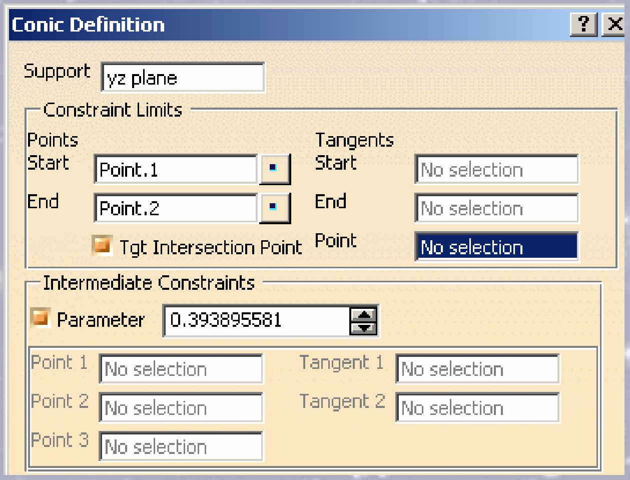
When $\lambda = ½$, the denominator become constant

If $\lambda=0$ one line and $\lambda=1$; two lines

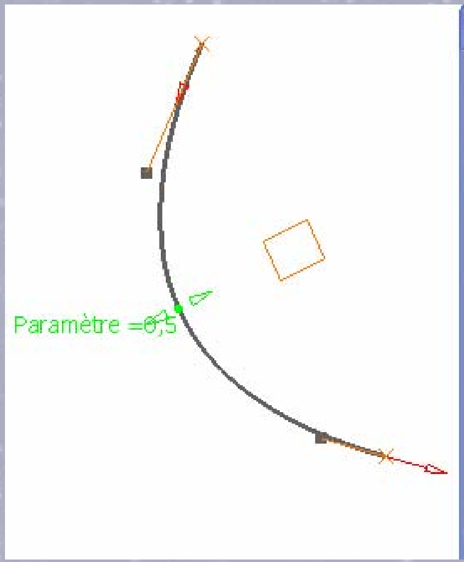
0 < Parameter ω < 0,5 → Ellipse

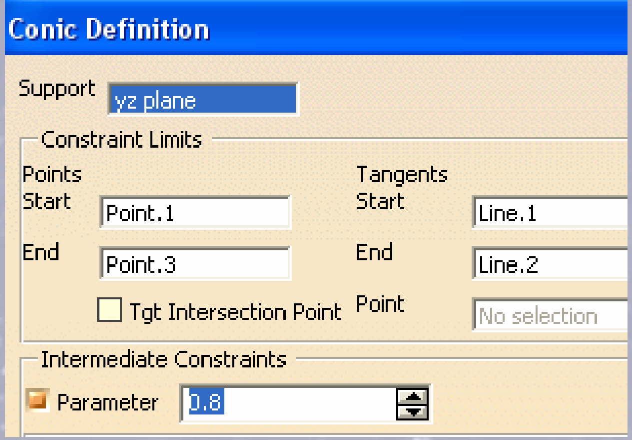
Parameter ω = 0,5 → Parabola

0,5 < Parameter ω < 1 → Hyperbola

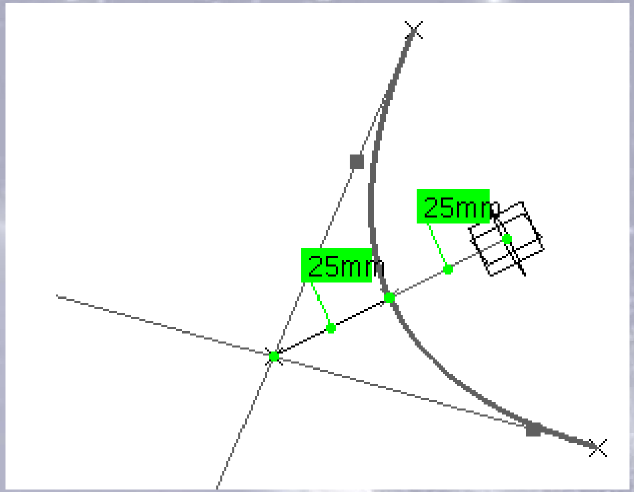


$$\overrightarrow{OI} = (1-\omega)\overrightarrow{OH} + \omega\overrightarrow{OP_1}$$

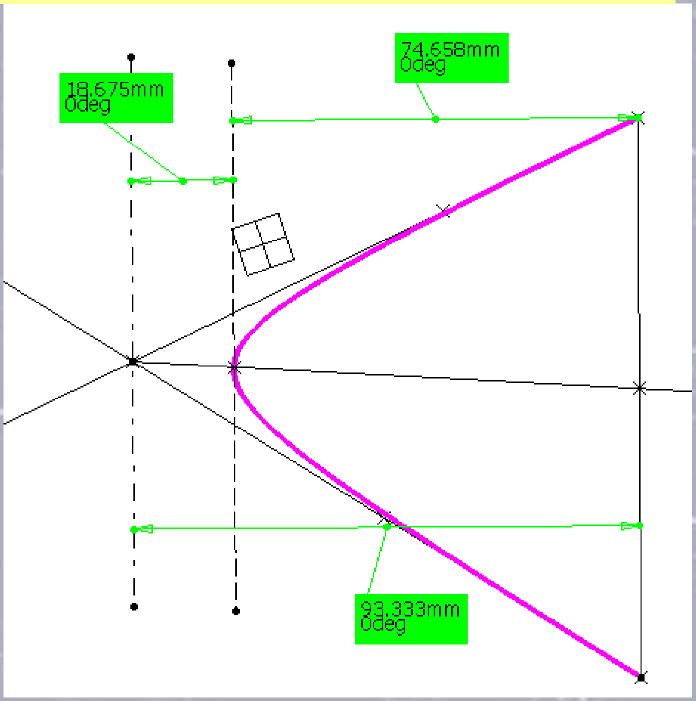




$$93{,}333 \times 0{,}8 = 74{,}664 \;!!!$$



ω = Linear interpolation between H and P_1



Basic-splines

Polynomial parametric shape per pieces → change of the position of a control point will have a limited impact on the function obtained.

Definition : A **B-spline** curve **of degree d (ou d'ordre k = d + 1)** is thus defined by:

- a **knots vector** $T = (t_0, t_1, \ldots, t_m)$, m+1 réels t_i
- n+1 **control points** P_i
- n+1 **weight functions** $N_{i,k}$ defined recursively on intervals $[t_i, t_{i+1}]$:

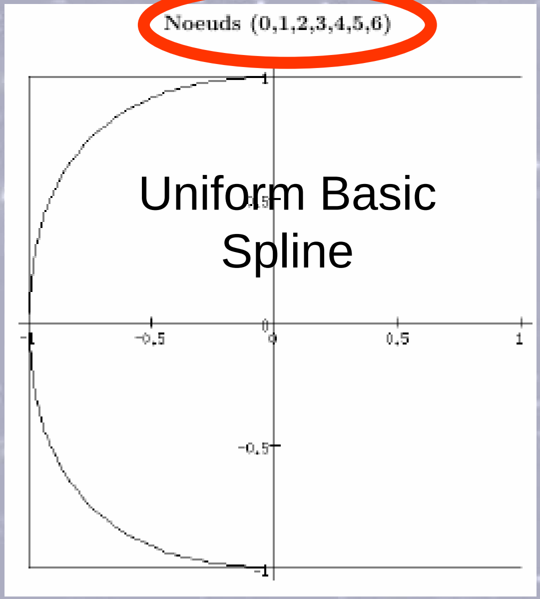
$$\overrightarrow{OM(u)} = \sum_{i=0}^{n} N_{i,d}(u)\overrightarrow{OP_i}$$

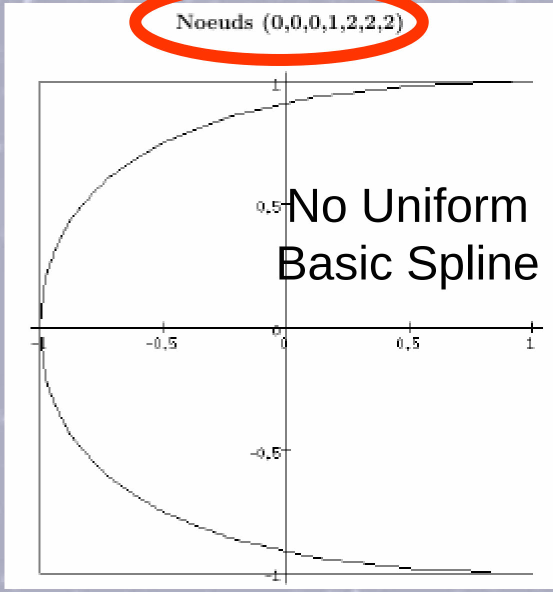
$$N_{i,1}(t) = \begin{cases} 1 & \text{if } u \in [t_i, t_{i+1}) \\ 0 & \text{otherwise} \end{cases}$$

$$N_{i,k}(t) = \left(\frac{t - t_i}{t_{i+k-1} - t_i}\right) N_{i,k-1}(t) + \left(\frac{t_{i+k} - t}{t_{i+k} - t_{i+1}}\right) N_{i+1,k-1}(t)$$

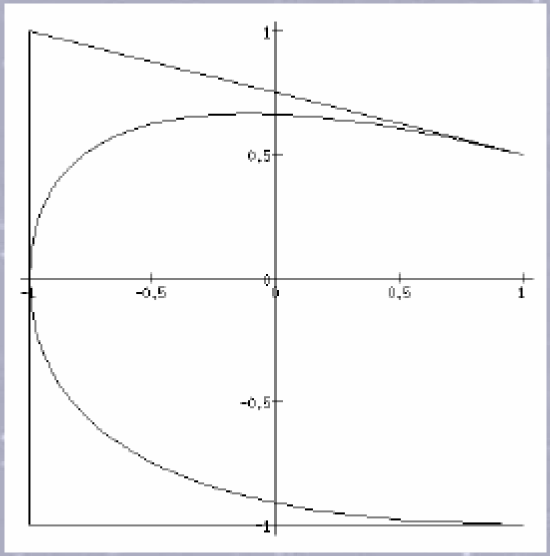
To build a B-spline curve of degree d from n+ 1 points P_i, it is thus necessary to be given m + 1 knots or m = n + d + 1, allowing to define the basic functions $N_{i,d}(u)$.

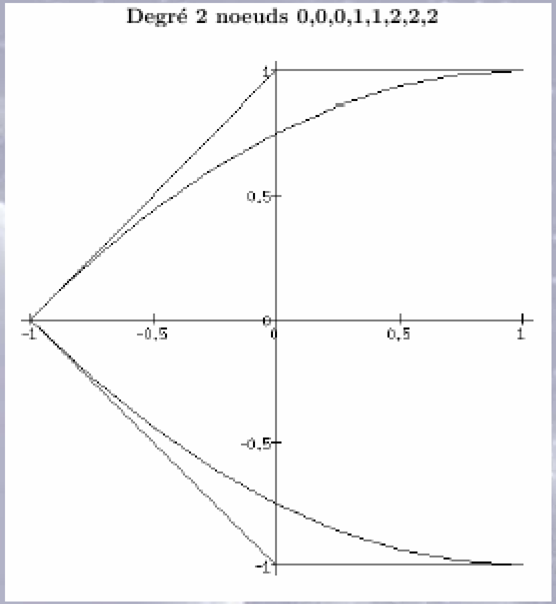
Example 1

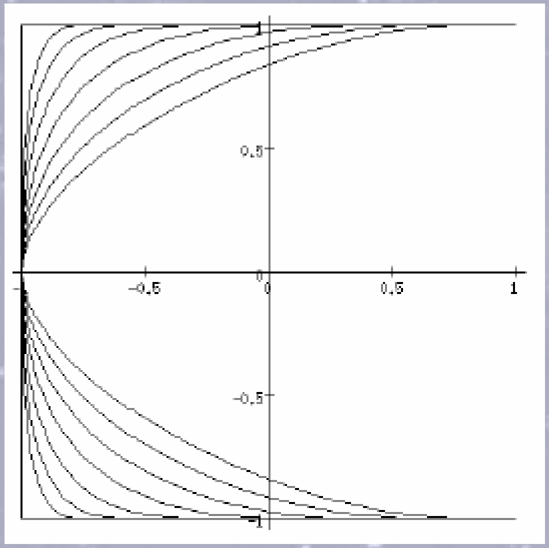




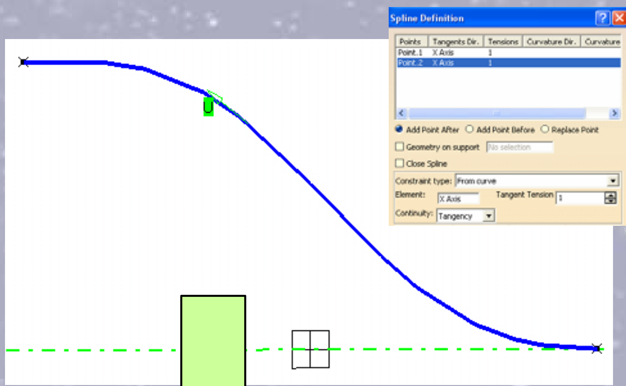
Displacement of control point







NUPBS CATIA

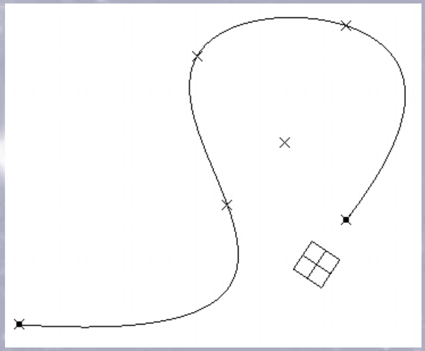


Geometric Analysis

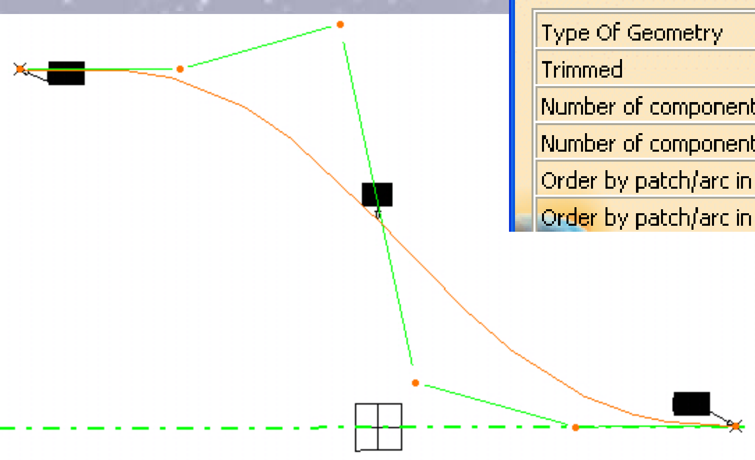
Type Of Geometry	SplineCurve
Trimmed	No
Number of components U	1
Number of components V	-
Order by patch/arc in U	-
Order by patch/arc in V	-

Geometric Analysis

Type Of Geometry	SplineCurve
Trimmed	No
Number of components U	4
Number of components V	-
Order by patch/arc in U	-
Order by patch/arc in V	-



Assistant de conversion



Geometric Analysis

Type Of Geometry	NupbsCurve
Trimmed	No
Number of components U	1
Number of components V	-
Order by patch/arc in U	6
Order by patch/arc in V	-

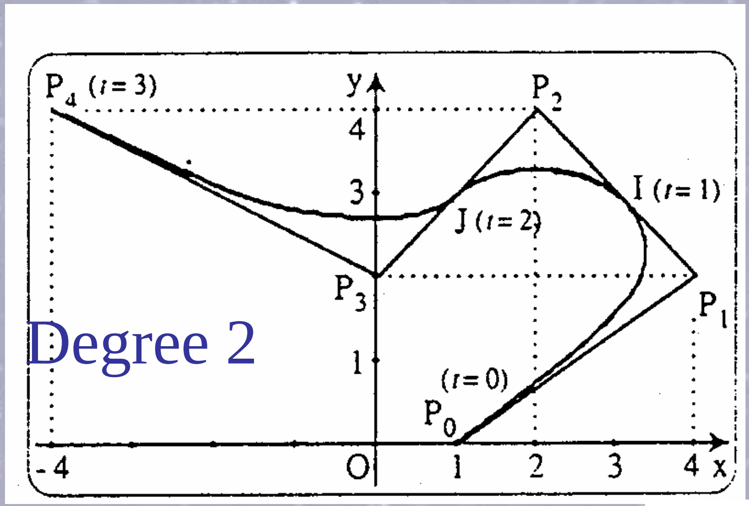
Geometric Analysis

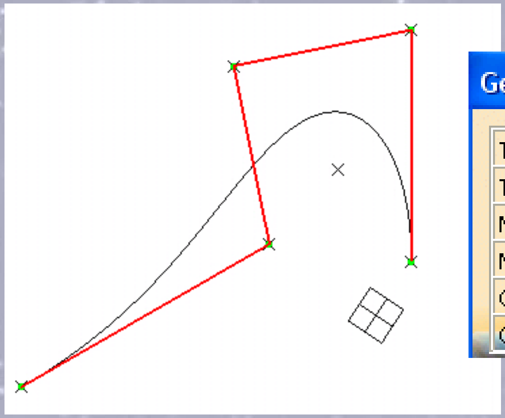
Type Of Geometry	NupbsCurve
Trimmed	No
Number of components U	4
Number of components V	-
Order by patch/arc in U	6
Order by patch/arc in V	-

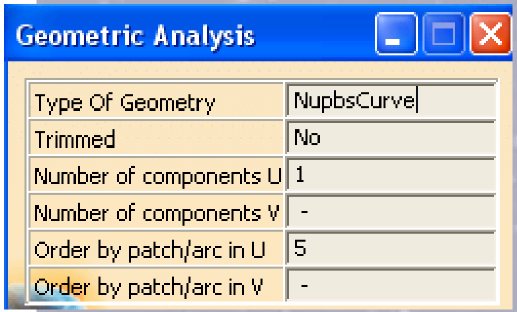
By defect, the degree of the curves created by Catia is **5**

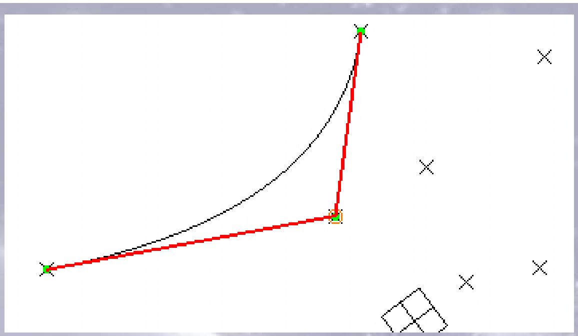
ordre k = d + 1

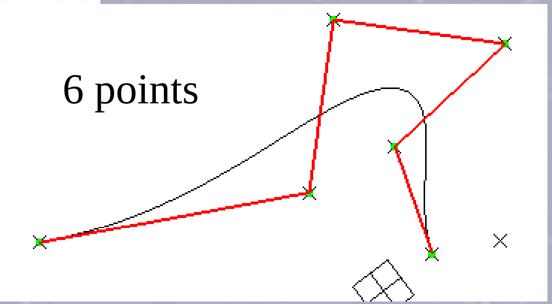
NUPBS CATIA

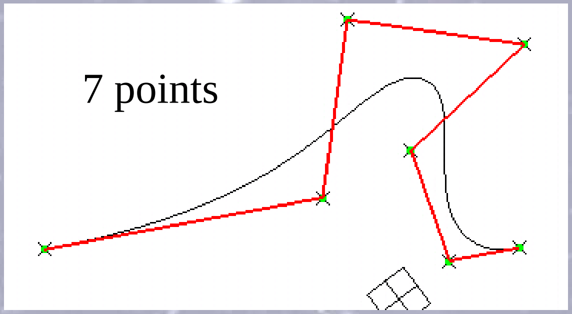

P4 (t = 3)
y
P2
4
3
I (t = 1)
J (t = 2)
P3
P1
Degree 2
1
(t = 0)
P0
-4
O
1
2
3
4
x

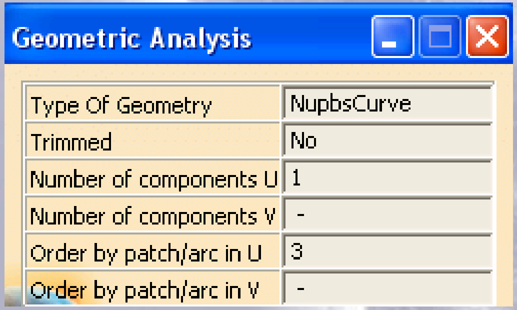


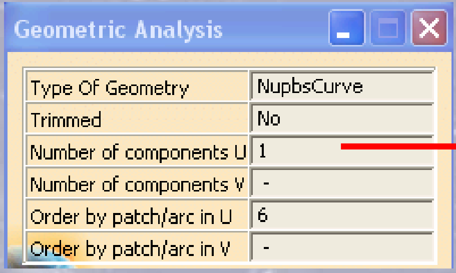

Geometric Analysis
Type Of Geometry | NupbsCurve
Trimmed | No
Number of components U | 1
Number of components V | -
Order by patch/arc in U | 5
Order by patch/arc in V | -

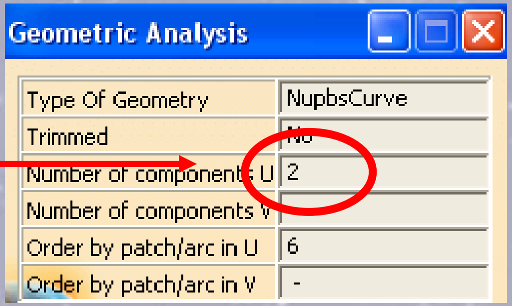



6 points


7 points


Geometric Analysis
Type Of Geometry | NupbsCurve
Trimmed | No
Number of components U | 1
Number of components V | -
Order by patch/arc in U | 3
Order by patch/arc in V | -

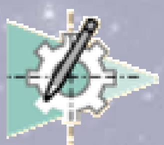

Geometric Analysis
Type Of Geometry | NupbsCurve
Trimmed | No
Number of components U | 1
Number of components V | -
Order by patch/arc in U | 6
Order by patch/arc in V | -


Geometric Analysis
Type Of Geometry | NupbsCurve
Trimmed
Number of component U | 2
Number of components V
Order by patch/arc in U | 6
Order by patch/arc in V | -

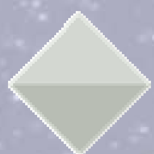
Where are NURBS in Catia V5

workshop DOCUMENTATION

PowerFit creates a ***NURBS*** surface



Creation of 3D ***NURBS*** curves



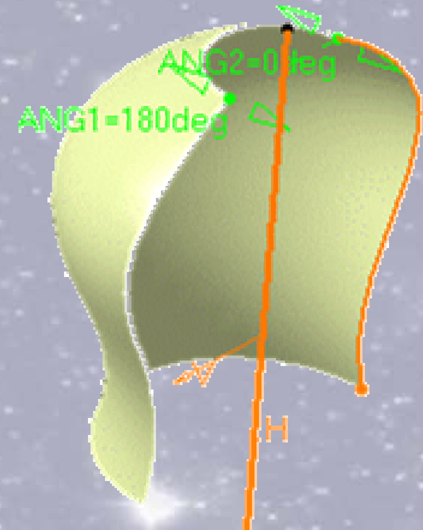
NURBS Formats in APT Output

Réponses DS :

« Elles sont supportées dans CATIA mais on n'offre pas de moyen d'en créer en interactif. Quand elles existent, elles proviennent d'un import »

« L'interactif ne permet pas de travailler avec un degré inférieur à 5 ni de choisir l'espacement paramétrique entre 2 noeuds (en V4 on pouvait faire tout ça) »

SIMPLE SURFACES



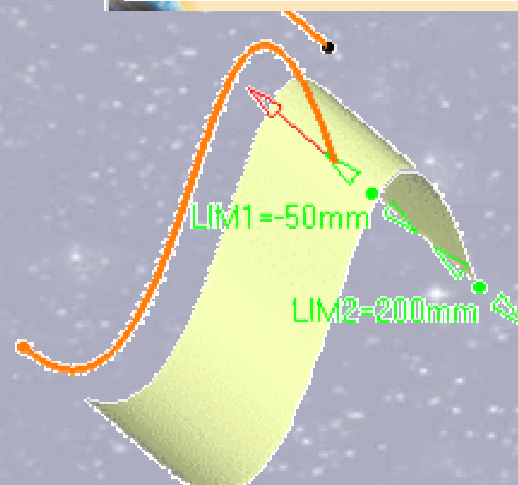
The C(v) curves are lines, circles,polynomials or others which are used for the generation of surfaces

Geometric Analysis

Type Of Geometry	RevolutionSurface
Trimmed	Yes
Number of components U	1
Number of components V	-
Order by patch/arc in U	-
Order by patch/arc in V	-

Revolution of a profile around an axis

$$\vec{S}_{rev}(u,v) = C(v)\ (\cos U\ \vec{e_1} + \sin U\ \vec{e_2}) + v\ \vec{e_3} + \vec{OA}$$

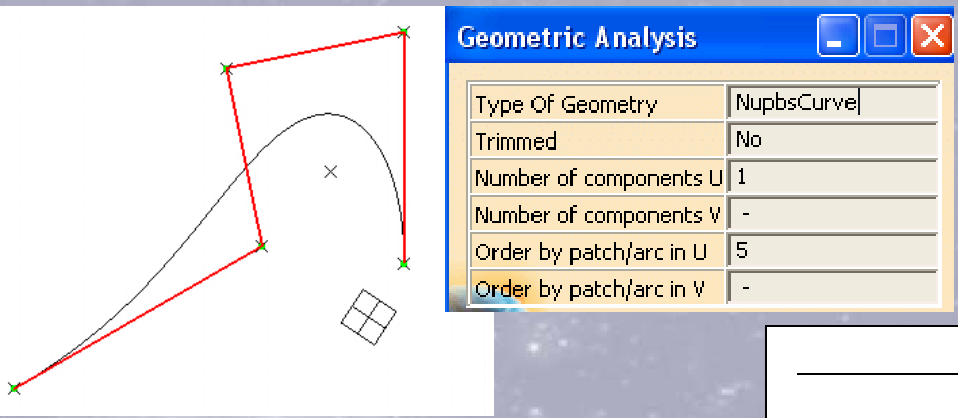


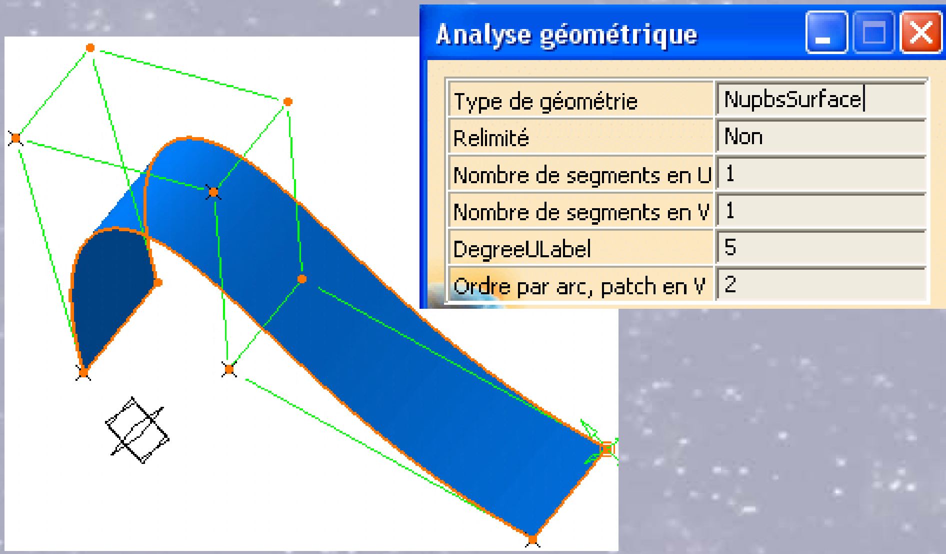
Extrusion of a profile in a given direction.

$$\vec{S}_{\text{éti}}(u,v) = \vec{C_1}(u) + v\ \vec{e3} + \vec{OPo}$$

NUPBS CATIA

Extruded surfaces





A curve extrude in the direction of the unit vector $\vec{n}$, through a distance Δ

$$\overrightarrow{OM}(u,v) = \sum_{i=0}^{n}\sum_{j=0}^{1} N_{i,d}(u)N_{j,1}(v)\overrightarrow{OP_{i,j}}$$

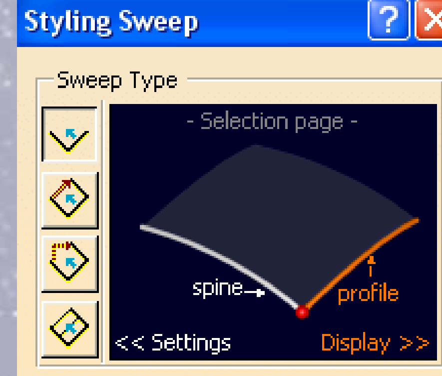
In the v-direction, controls points

$\overrightarrow{OP}_{i,0} = \overrightarrow{OP}_i$ and $\overrightarrow{OP}_{i,1} = \overrightarrow{OP}_i + \Delta\,\vec{n}$

Knots vector

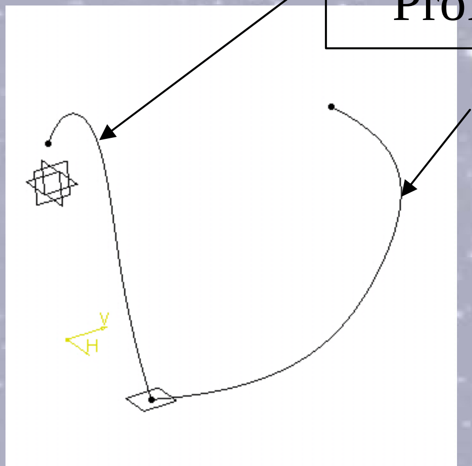
$(v_0, v_1, v_2, v_3) = (0, 0, 1, 1)$

Swept surface



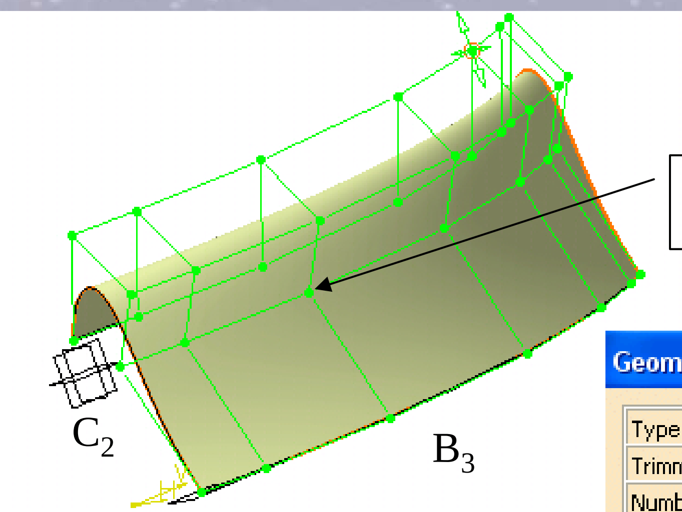
Profil C(u) degree d

Guide B(v) degree e



NUPBS

$$\overrightarrow{OM(u,v)} = \sum_{i=0}^{n}\sum_{j=0}^{p} N_{i,d}(u)N_{j,e}(v)\overrightarrow{OP_{i,j}}$$



Controls points

$$\overrightarrow{OP_{i,j}} = \overrightarrow{OC_i} + \overrightarrow{OB_j}$$

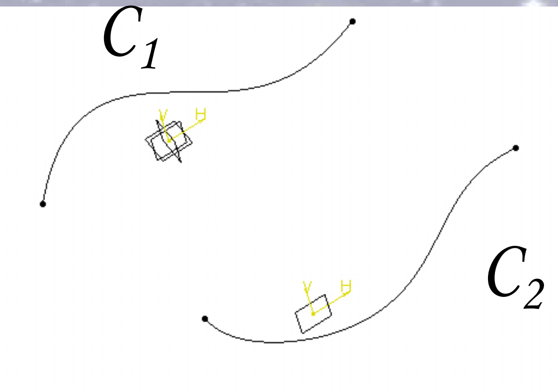
Geometric Analysis

Type Of Geometry	NupbsSurface
Trimmed	No
Number of components U	1
Number of components V	4
Order by patch/arc in U	5
Order by patch/arc in V	12

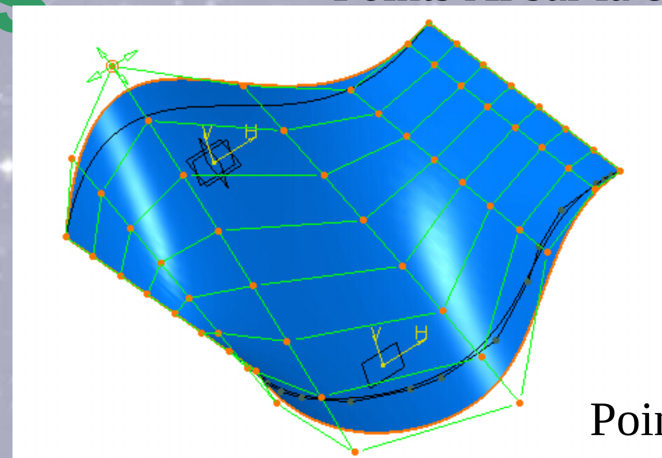
NURBS

$$\begin{aligned} X(u,v) &= g(u) + p(v) \\ &= \frac{\sum_i B_{i,k}(u) w_i P_i}{\sum_i B_{i,k}(u) w_i} + \frac{\sum_j B_{j,k'}(v) w'_j Q_j}{\sum_j B_{j,k'}(v) w'_j} \\ &= \frac{\sum_{i,j} B_{i,k}(u) B_{j,k'}(v) w_i w'_j (P_i + Q_j)}{\sum_{i,j} B_{i,k}(u) B_{j,k'}(v) w_i w'_j}. \end{aligned}$$

Ruled Surfaces



Points Ai sur la courbe C_1



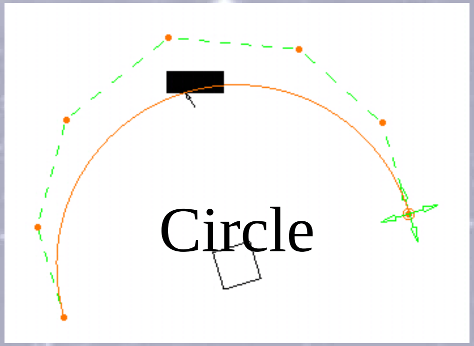
Points Bi sur la courbe C_2

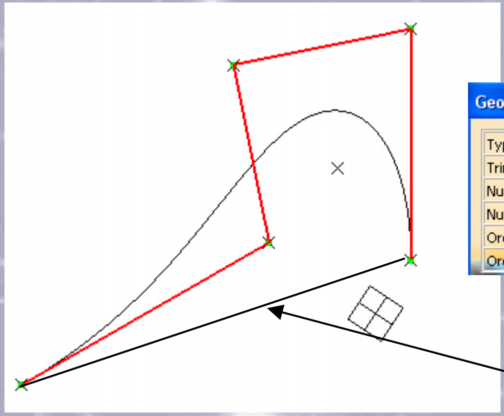
$$\overrightarrow{OM}(u,v) = \sum_{i=0}^{n}\sum_{j=0}^{1} N_{i,d}(u) N_{j,1}(v) \overrightarrow{OP_{i,j}}$$

Controls points $\overrightarrow{OP_{i,0}} = \overrightarrow{OA_i}$ $\overrightarrow{OP_{i,1}} = \overrightarrow{OB_i}$

The specified curves C_i have the same degree and the same knot vector

Surfaces of revolution





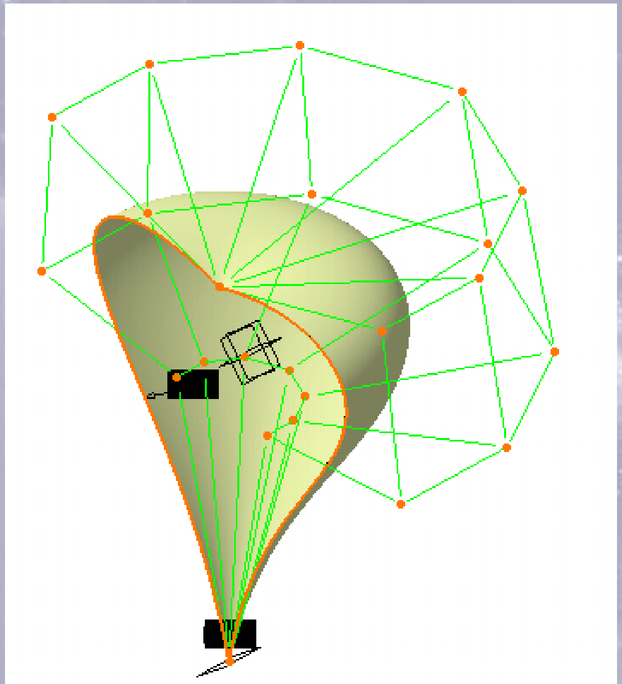
Axe

Geometric Analysis

Type Of Geometry	NupbsCurve
Trimmed	No
Number of components U	1
Number of components V	-
Order by patch/arc in U	5
Order by patch/arc in V	-

Analyse géométrique

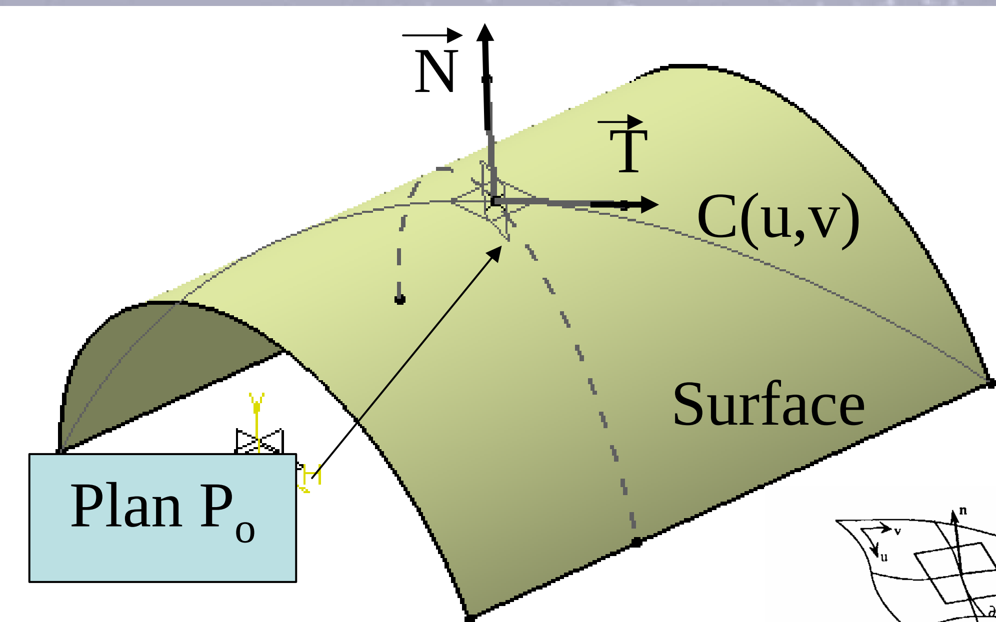
Type de géométrie	PNupbs
Relimité	Non
Nombre de segments en U	1
Nombre de segments en V	-
DegreeULabel	7
Ordre par arc, patch en V	-



Geometric Analysis

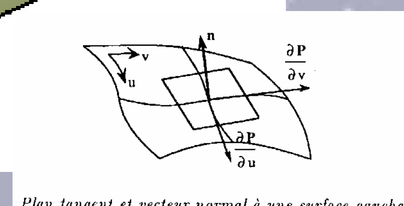
Type Of Geometry	NupbsSurface
Trimmed	No
Number of components U	1
Number of components V	1
Order by patch/arc in U	5
Order by patch/arc in V	7

Analyze the surfacique curvature



k_n normal curvature on 1 point

$$\frac{d^2\overrightarrow{C(u,v)}}{ds^2} \bullet \overrightarrow{N} = k_n$$



Plan tangent et vecteur normal à une surface gauche.

$\overrightarrow{N}$: normal vector of the surface

$$\overrightarrow{N} = \frac{\dfrac{\partial \overrightarrow{C}}{\partial u} \wedge \dfrac{\partial \overrightarrow{C}}{\partial v}}{\left\| \dfrac{\partial \overrightarrow{C}}{\partial u} \wedge \dfrac{\partial \overrightarrow{C}}{\partial v} \right\|}$$

$$k_n = \frac{Ldu^2 + 2Mdudv + Ndv^2}{Edu^2 + 2Fdudv + Gdv^2}$$ with

$$E = \frac{\partial \overrightarrow{C}}{\partial u} \bullet \frac{\partial \overrightarrow{C}}{\partial u} \qquad F = \frac{\partial \overrightarrow{C}}{\partial u} \bullet \frac{\partial \overrightarrow{C}}{\partial v} \qquad G = \frac{\partial \overrightarrow{C}}{\partial v} \bullet \frac{\partial \overrightarrow{C}}{\partial v}$$

$$L = \frac{\partial^2 \overrightarrow{C}}{\partial u^2} \bullet \overrightarrow{N} \qquad M = \frac{\partial^2 \overrightarrow{C}}{\partial u \partial v} \bullet \overrightarrow{N} \qquad N = \frac{\partial^2 \overrightarrow{C}}{\partial v^2} \bullet \overrightarrow{N}$$

Evolution of the curvature k_n for the curves $\overrightarrow{C(u,v)}$ on the surface according to (du/dv) when the plan P_o containing the normal $\overrightarrow{N}$ carry out a rotation around N.

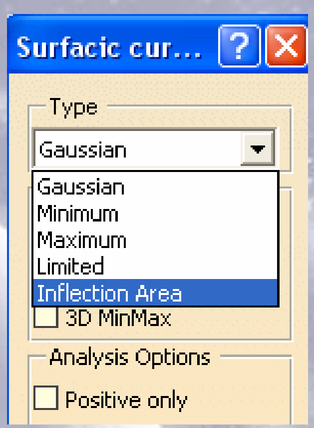
Solutions

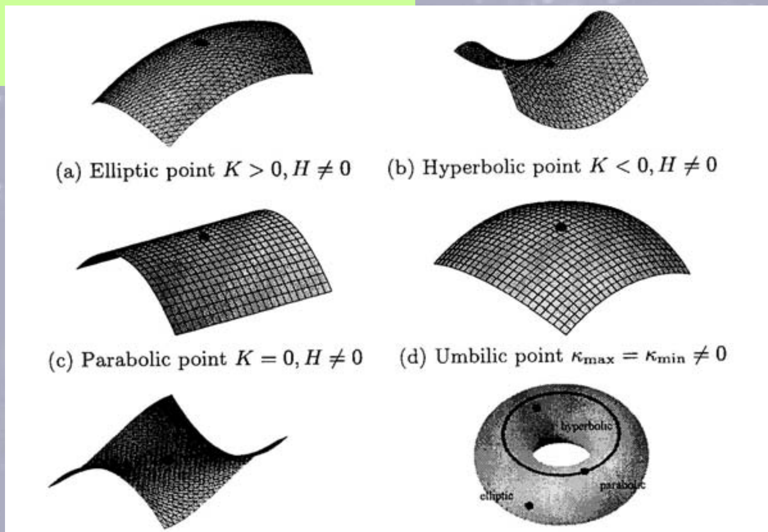
The extremum values of k_N are solution of :

$$(EG - F^2)k_n^2 - (EN + GL - 2FM)k_n + LN - M^2 = 0$$

$$2H = k_{n\,\min} + k_{n\,\max} = \frac{EN + GL - 2FM}{EG - F^2} = mean\_curvature$$

$$K = k_{n\,\max} . k_{n\,\min} = \frac{LN - M^2}{EG - F^2} = Gaussian\_curvature$$



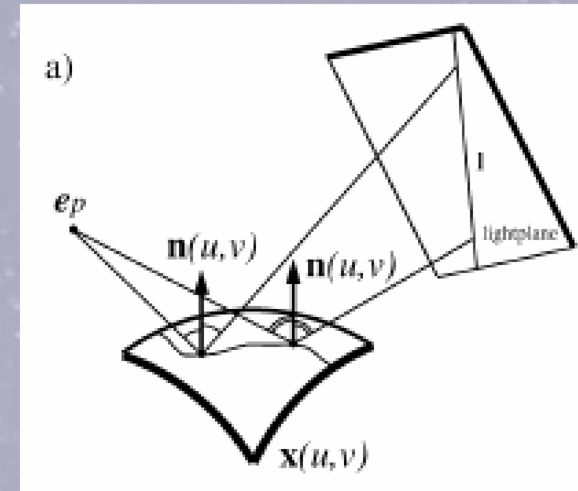


(a) Elliptic point $K > 0, H \neq 0$ (b) Hyperbolic point $K < 0, H \neq 0$

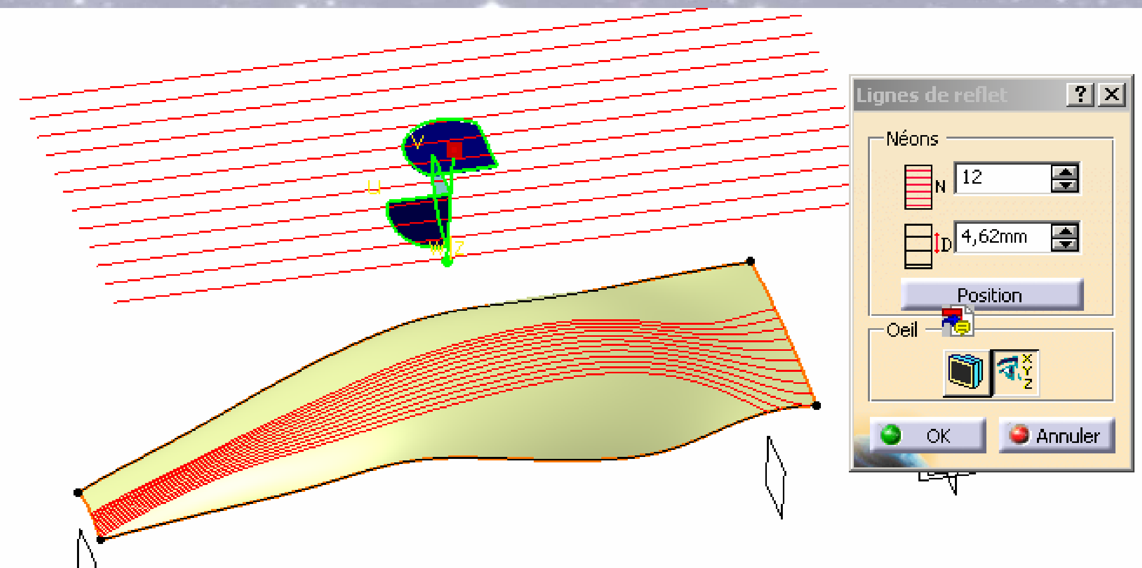
(c) Parabolic point $K = 0, H \neq 0$ (d) Umbilic point $\kappa_{\max} = \kappa_{\min} \neq 0$

(e) Planar point $\kappa_{\max} = \kappa_{\min} = 0$ (f) Elliptic, hyperbolic, and parabolic points on a torus

Analysis with reflect lines



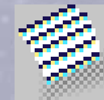
A reflection line on the surface X(u,v) is the mirror image of the light line L on X(u,v) while looking from fixed eye e*p*.

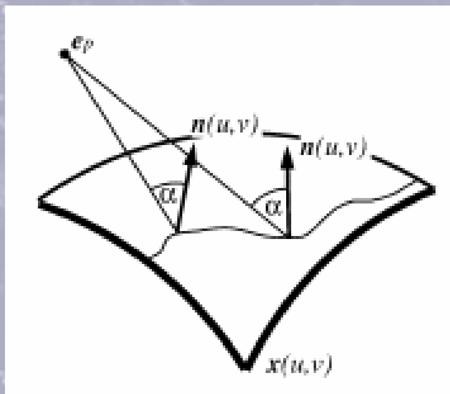


X(u,v) surface parametrized
N(u,v) normal vector of the surface

It shows the geometry for a point on the surface where an observer sees a reflection. For such a point, the display program would then assign a bright white color to the surface. For points where a reflection is not seen, the standard color of the surface is chosen. This color is darker than the white reflection color. The simulation now determines the correct color value for a large number of points on the surface

Analysis with isophotes



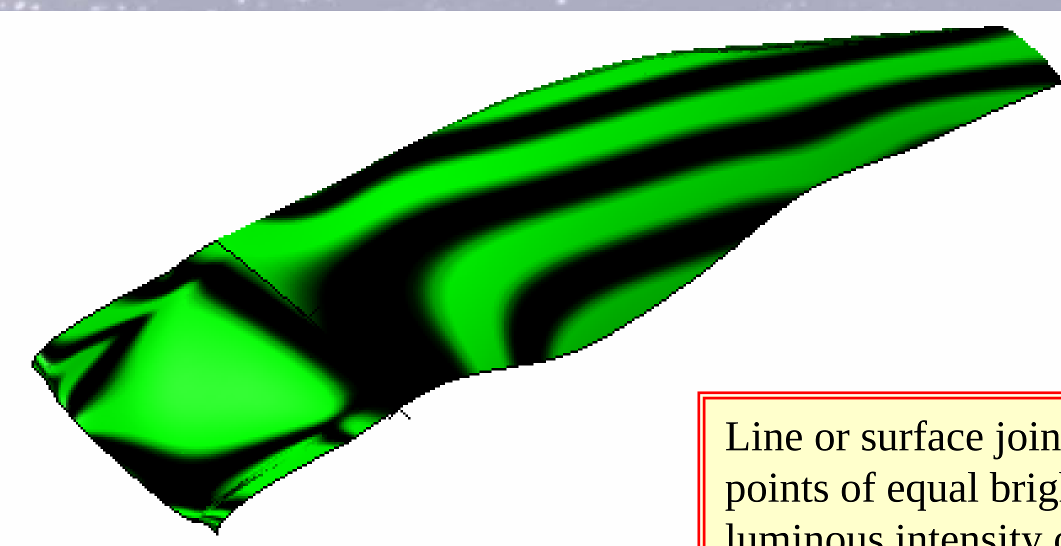


Definition of isophotes using an eye direction e_p : all points with a constant angle α between e_p and the normal $\vec{N}(u,v)$ lie on an isophote.

Light direction vector = $\vec{r}$

$$\vec{r} \cdot \vec{N}(u,v) = \text{Constant}$$

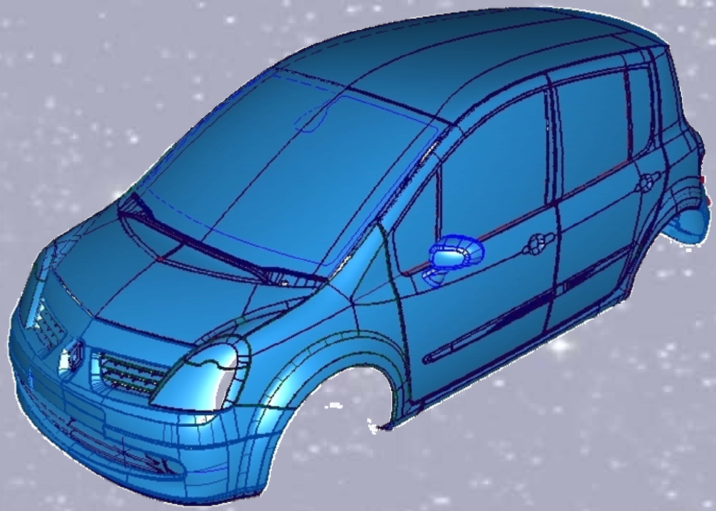
Isophote depends only on normal, not position

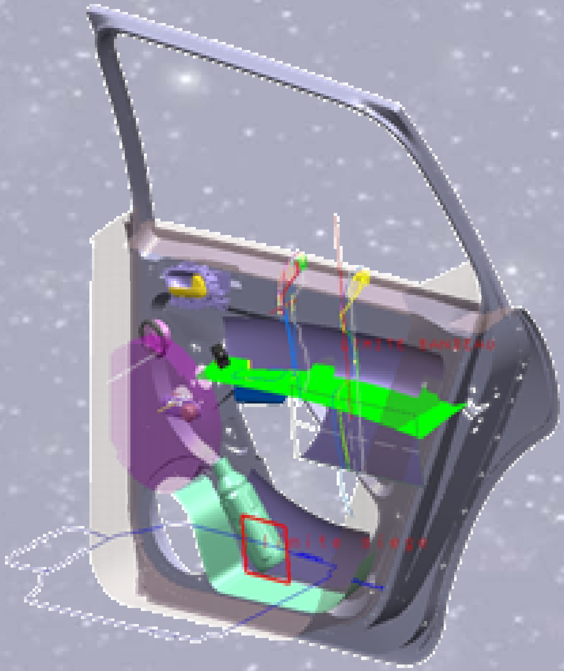


Line or surface joining the points of equal brightness or luminous intensity of a given source

CONCLUSIONS

- Niveau mathématique insuffisant des étudiants Bac +3 pour suivre jusqu'au bout,
- Encore certains points à développer :
  - NUPBS,
  - Cohérence entre les calculs et la modélisation
  - Analyses
- Fiabilité de la documentation DS !
- Plus facile en V4
- Freestyle à orienter « Plan de forme »





MERCI POUR VOTRE ATTENTION

QUESTIONS ??