Student Notes:

CATIA V5 Training
Foils

Introduction to the Mathematical Concepts of CATIA V5

Version 5 Release 19
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EDU_CAT_EN_MTH_FF_V5R19

Student Notes:

About this course

Objectives of the course

Upon completion of this course you will be able to:

- Understand the mathematical concepts for curve and surface definition in CATIA V5.

Targeted audience

GSD and/or FreeStyle users

Prerequisites

Students attending this course must have knowledge of GSD and FreeStyle Fundamentals

4 hours

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The Feature Approach (1/2)

- **CATIA V5 supports a FEATURE APPROACH.**
- **It means that users create and handle objects which are more than mathematical objects because they carry more than just mathematical definitions.**
- **The mathematical definition of the object is no more than one of the representations of the feature which CATIA may refer to when needed.**
  - **For example, another representation of a surface in CATIA V5 is its triangular mesh used for shaded display or draft analysis.**

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The Feature Approach (2/2)

FEATURE

=

ASSOCIATIVITY INFORMATION + **ATTRIBUTES** + **MATHEMATICAL INFORMATION**

ASSOCIATIVITY INFORMATION

-parents

-creation operator

-children

…

ATTRIBUTES

-display attributes:
color,
layer,
visibility,
…

-applicative attributes:
material,
physical properties,
tolerances
…

MATHEMATICAL INFORMATION

-geometrical information
equations, points,
vectors
…

-topological information
vertices, edges, faces,
orientations,
…

Note: a Datum is a feature with no parents nor creation operator in the asociativity information (it may only have children)

The Mathematical Level

- **The mathematical part of the object definition includes both geometry and topology.**
  - **The geometry defines the shape itself and its location in space,**
  - **The geometry is defined by mathematical objects such as points, vectors, angles, polynomials, …**
  - **The topology ensures the consistent assembly of the geometrical elements (connections, orientations)**
  - **It is defined by mathematical objects such as vertices, edges, faces**

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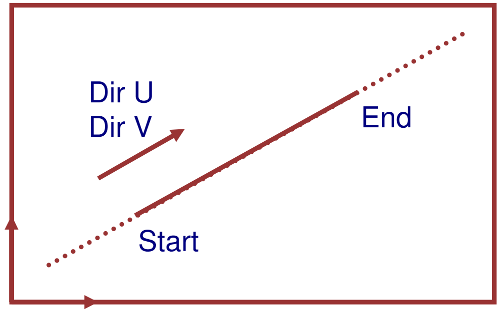
The Geometry Level for Curves (1/6)

- **Curves are described by canonic or parametric forms**
- **Examples of canonic forms**



Line defined by:

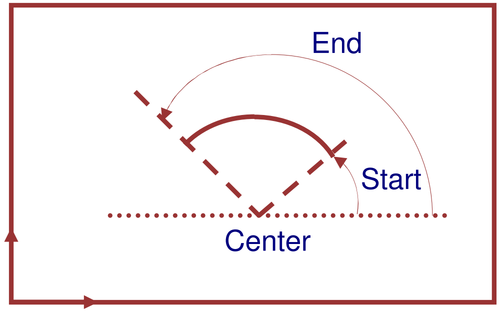
-An underlying surface (may be a plane)

-An origin point

-A direction on underlying surface

-A start position

-An end position



Circle defined by:

-underlying surface

-center

-radius

-start angle

-end angle

Canonic forms
- **are compact (little data)**
- **are exact**
- **make it easy to benefit from the characteristics of the object (example: select a line to define a direction)**

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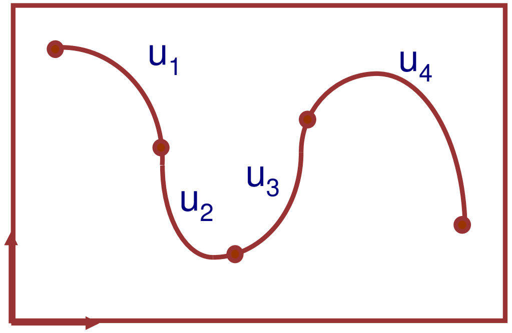
Student Notes:

The Geometry Level for Curves (2/6)

- **Other types of curves: parametric curves = NURBS**

NURBS = Non-Uniform Rational B-Spline

- **Non-Uniform**



A NURBS curve may be described by several arcs, or spans, or segments. Each segment is described by a parametric form: it has its own set of parametric representations, for example segment number i:

$X = F_{Xi}(u_i)$

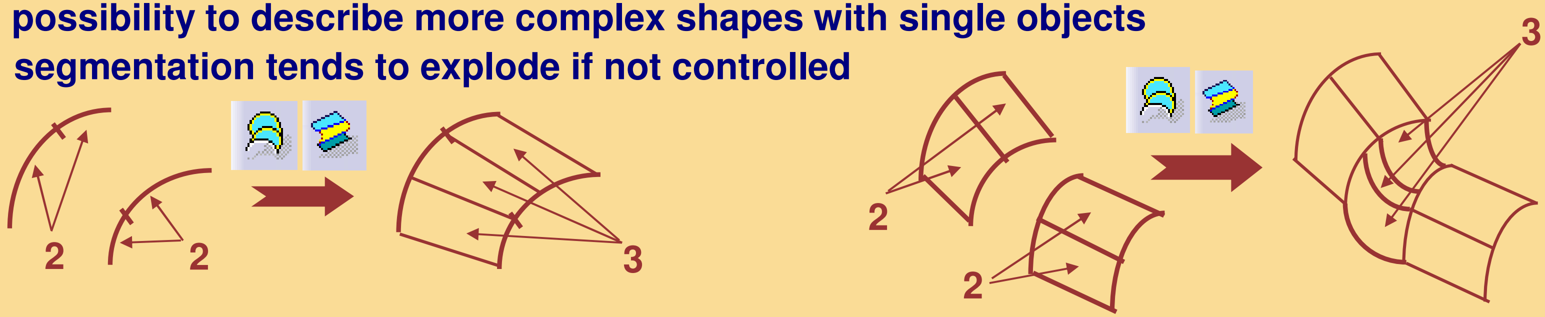
$Y = F_{Yi}(u_i)$

$Z = F_{Zi}(u_i)$

Note: the segments cannot be separated by the Disassemble command.

+ possibility to describe more complex shapes with single objects

- segmentation tends to explode if not controlled

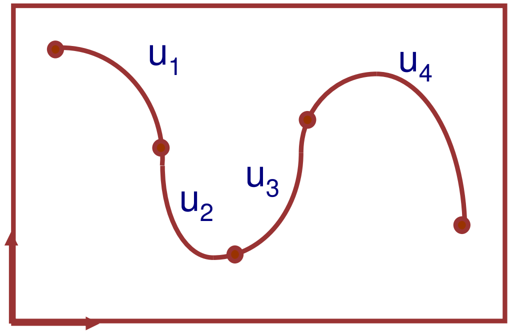


Student Notes:

The Geometry Level for Curves (3/6)

NURBS = Non-Uniform Rational B-Spline

- **Rational** Each segment is described by a rational form $X = F_{xi}(u_i) = \frac{P_{xi}(u_i)}{Q_{xi}(u_i)}$

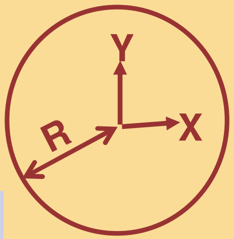


With $P_{xi}(u_i)$ and $Q_{xi}(u_i)$ being polymomials,
i.e. mathematical forms such as:
$P_{xi}(u) = A_0 + A_1.u + A_2.u^2 + \ldots + A_n.u^n$

NURBS created in CATIA V5 are usually polynomial, Q (u) =1
this is why they are called NUPBS (P for Polynomial)

+ possibility to describe exact conics, for example a circle can be given by:

$X = R\,\frac{1-u^2}{1+u^2}$ $\quad Y = R\,\frac{2u}{1+u^2}$ **… but canonic forms are also exact**



- degrees tend to explode if not controlled
example: ruled surface on two curves given by $\frac{P_1(u)}{Q_1(u)}$ **and** $\frac{P_2(u)}{Q_2(u)}$

$S(u,v) = (1-v)\frac{P_1(u)}{Q_1(u)} + v\frac{P_2(u)}{Q_2(u)} = \frac{(1-v)\,P_1(u)\,Q_2(u) + v\,P_2(u)\,Q_1(u)}{Q_1(u)\,Q_2(u)}$ **degrees of polynomials add**

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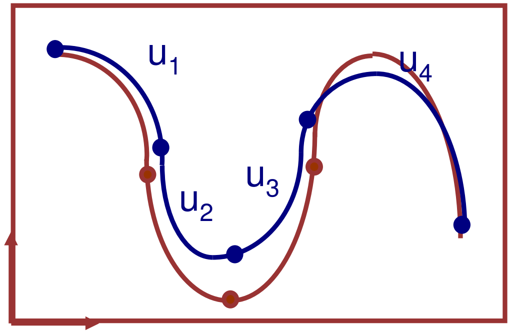
The Geometry Level for Curves (4/6)

NURBS = Non-Uniform Rational B-Spline

- **B-Splines**

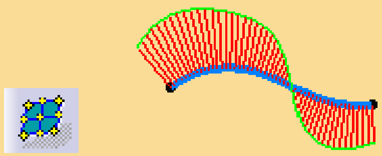


The definition of a B-Spline curves includes the description of the transitions between its segments.

Note: in CATIA V5, NURBS are always internally curvature continuous,
= transitions between segments are always C2.

+ possibility to safely manipulate complex objects, for example to deform complex curves while preserving their overall smoothness (no unexpected gap or sharp corner appearing)

- It may be difficult manipulate the curve while keeping it good looking (example: deform by control points while keeping a nice curvature distribution)



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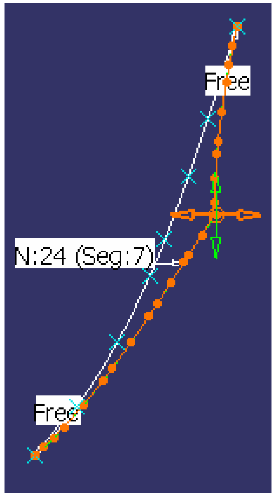
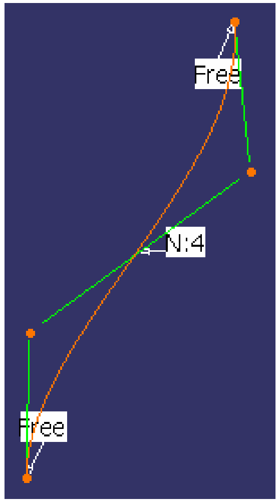
The Geometry Level for Curves (5/6)



Note 1: the polygonal representation

- **A NURBS can be represented by a polygon = a set of control points**
- **This representation is often used in style design for intuitive shaping**



Note 2: a special case of NURBS

- **A NURBS can be uniform (only one segment)**
- **It can also be polynomial ($Q_{xi}(u_i) = 1$)**

This type of curves is known as Bezier curve

It is favored by style designers because it is easier to manipulate (fewer points, well known properties)

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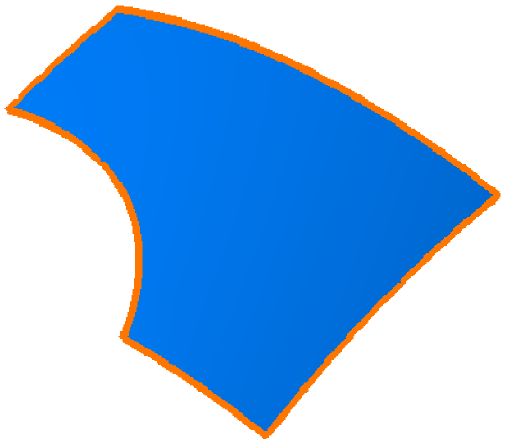
The Geometry Level for Curves (6/6)

General validity criteria for curves:

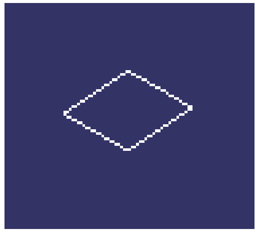
- **A mono-cell curve must be C2 continuous, i.e. mathematically curvature continuous.**
- **It means that if an action produces a curve that is not C2 continuous, it is cut at each discontinuity and the C2 pieces become cells which are called edges and are assembled in a topology.**
- **The topology consists in a list of edges with shared vertices (common to several edges) and free vertices (common to one edge only = end points).**

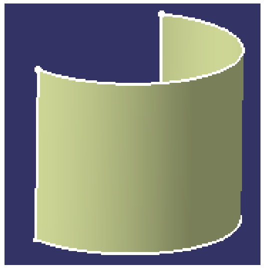
  - **Example: the boundary feature is a single CATIA curve which is not C2 continuous.**
  - **This CATIA curve is made of several C2 continuous curves called edges that are assembled by a topology (joined).**
  - **The edges may be isolated from each other by an Extract or a Disassemble command (option All Cells).**

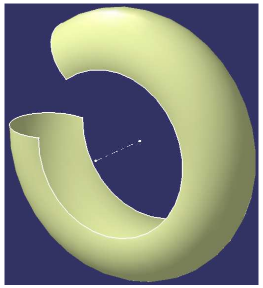


The Geometry Level for Surfaces (1/5)

- **Surfaces can also described by canonic or parametric forms**
- **Some surfaces can also be described by their creation process**
- **Since R14 CATIA V5 also handles subdivision surfaces**
- **Examples of canonical surfaces:**
  - **Plane**
  - **Cone**
  - **Sphere**
  - **Cylinder**
  - **Torus**







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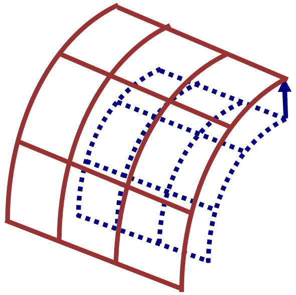
The Geometry Level for Surfaces (2/5)

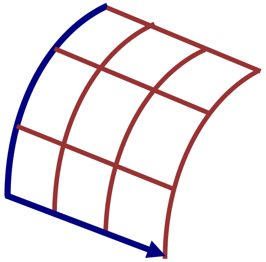
- **Procedural surfaces**
  - **A procedural surface is described by a creation process and the corresponding input**

Examples of procedural surfaces:

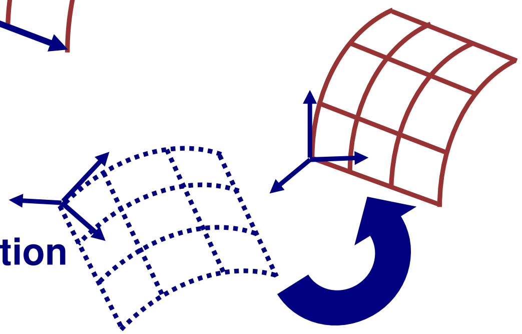
- **Offset surface**
  defined by a surface + a distance



- **Tabulated cylinder**
  defined by a curve, a direction, two lengths



- **Linear transformation surface**
  defined by a surface and a geometric transformation



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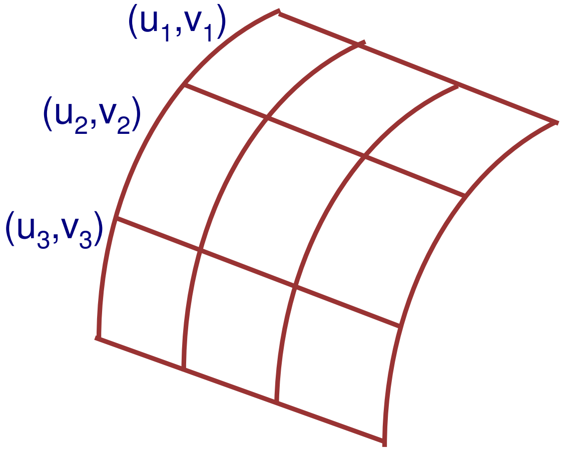
The Geometry Level for Surfaces (3/5)

- **Parametric surfaces = NURBS**



The definition for surfaces is similar to the definition for curves with 2 parameters: surfaces may be described by several segments (Non Uniform), each segment is described by a rational form (Rational), but surfaces can be handled globally thanks to B-Spline techniques.

$$X = F_{Xi}(u_i,v_i) = \frac{P_{xi}(u_i,v_i)}{Q_{xi}(u_i,v_i)}$$

$$Y = F_{Yi}(u_i,v_i)$$

$$Z = F_{Zi}(u_i,V_i)$$

Notes:
- NURBS created in CATIA V5 are usually (almost always) polynomials (NUPS)
- they are always curvature continuous (C2),
- NURBS surfaces can be represented and handled by control points,
- uniform polynomial NURBS are known as Bezier patches

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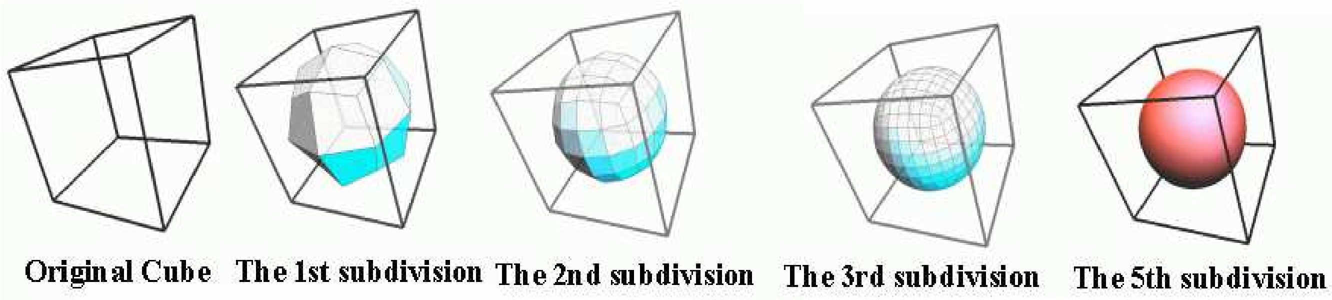
The Geometry Level for Surfaces (4/5)

Imagine & Shape

Subdivision surfaces

Subdivision is an algorithmic technique to generate smooth surfaces as a sequence of successively refined polyhedral meshes.

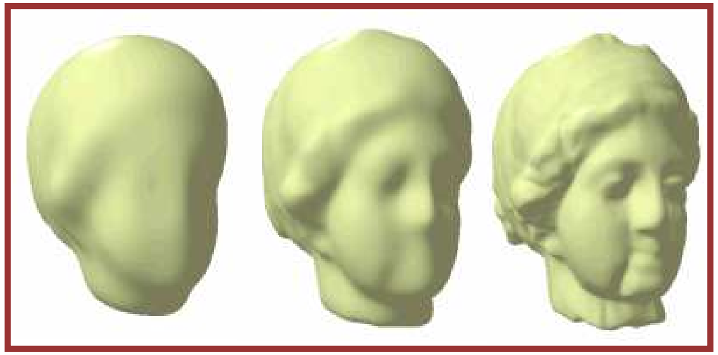


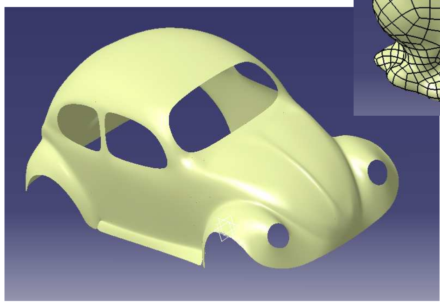
Advantages:

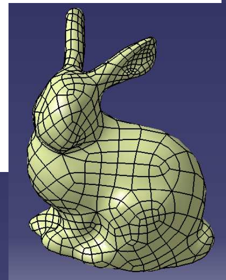
A complex object can be represented with only one multi-faced surface

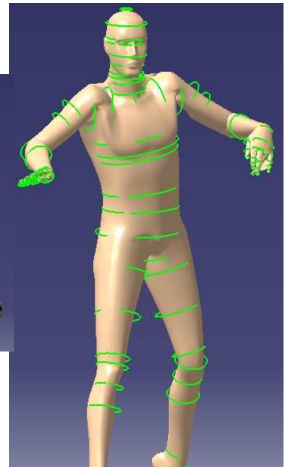
The surface is refined only where required (details)

=> Easy manipulation + Data size reduced







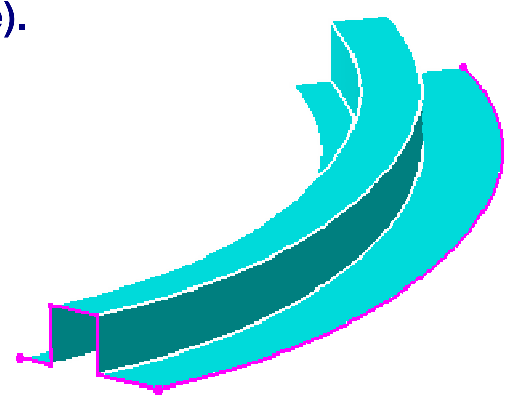


Student Notes:

The Geometry Level for Surfaces (5/5)

General validity criteria for surfaces:

- **A mono-cell surface must be C2 continuous.**
- **It means that if an action produces a result that is not C2 continuous, it is cut at each discontinuity and the C2 pieces become cells which are called faces and are assembled in a topology.**
  - **Example: the sweep feature is a single surface which is not C2 continuous (not even C1 in this case). It is made of several C2 continuous faces that are assembled by a topology (joined). The geometric surfaces may be isolated from each other by a Disassemble command (option All Cells).**



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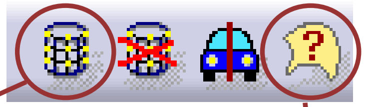
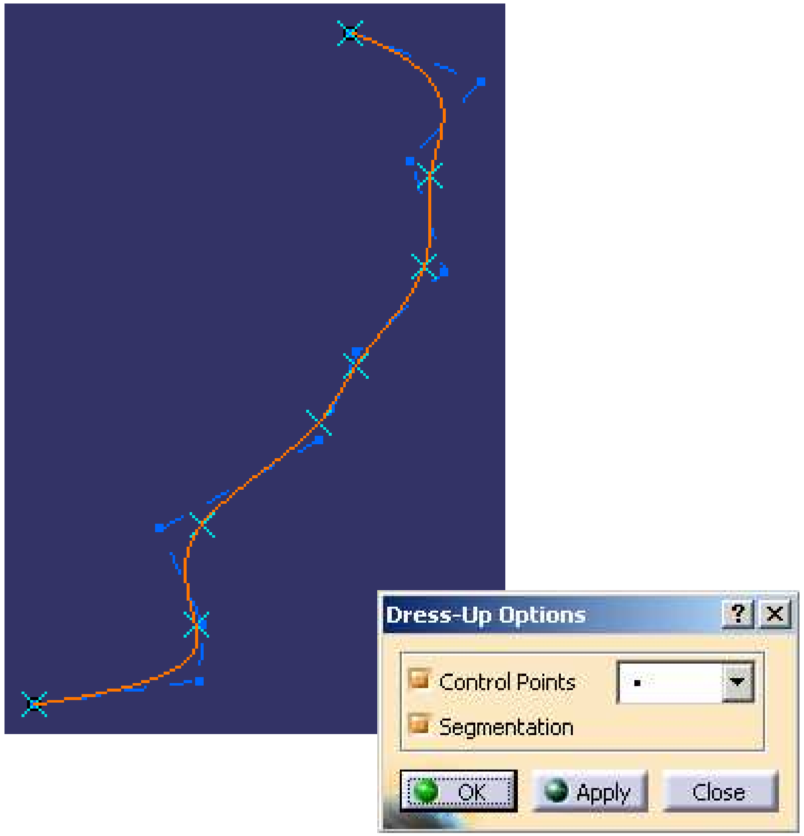
Object Analysis



- **FreeStyle offers tools to analyze objects.**

Dress-up options: applicable to NURBS curves and surfaces only



Geometry analysis: applicable to all curves and surfaces